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Screening and Adverse Selection in Frictional Markets*

Online Appendix

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Appendices

*The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

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A Omitted Proofs

This section contains proofs of the results presented in the main text.

A.1 Proofs from Section 3

A.1.1 Proof of Lemma 1

Proof. Both results are similar to existing results (see, for example, Dasgupta and Maskin (1986)), and thus we keep the exposition brief. To establish that $x_l = 1$ in all equilibrium menus, suppose by way of contradiction that some equilibrium menu $\mathbf{z} = (\mathbf{z}_l, \mathbf{z}_h)$ has $x_l < 1$ and $t_l \in \mathbb{R}_+$, yielding a low-quality seller utility u_l. Now, consider a deviation $\mathbf{z}' = (\mathbf{z}'_l, \mathbf{z}_h)$ with $x'_l = x_l + \epsilon$ for $\epsilon \in (0, 1 - x_l]$ and $t'_l = t_l + \epsilon c_l$. Note that $u'_l = u_l$, so that $\mathbf{z}_l$ and $\mathbf{z}'_l$ are accepted with the same probability, but

$$x_l v_l - t_l < x_l v_l - t_l + \epsilon(v_l - c_l) = x'_l v_l - t'_l,$$

so that $\mathbf{z}'_l$ earns the buyer a higher payoff when it is accepted, implying existence of a profitable deviation. Therefore, no equilibrium menu features $x_l < 1$.

To establish that a low-quality seller's incentive compatibility constraint binds in all equilibrium menus, suppose by way of contradiction that some equilibrium menu $\mathbf{z} = (\mathbf{z}_l, \mathbf{z}_h)$ has $t_l > t_h + c_l(1 - x_h)$. Now, consider a deviation $\mathbf{z}' = (\mathbf{z}_l, \mathbf{z}'_h)$ with $x'_h = x_h + \epsilon$ and $t'_h = t_h + \epsilon c_h$ for $\epsilon \in \left(0, \frac{t_l - t_h - c_l(1-x_h)}{c_h - c_l}\right]$, which is a nonempty interval by assumption. The upper bound on ϵ ensures that the incentive compatibility constraint on type l sellers is not violated. In addition, note that $u'_h = u_h$, so that $\mathbf{z}_h$ and $\mathbf{z}'_h$ are accepted with the same probability, but

$$x_h v_h - t_h < x_h v_h - t_h + \epsilon(v_h - c_h) = x'_h v_h - t'_h,$$

so that $\mathbf{z}'_h$ earns the buyer a higher payoff when it is accepted, implying existence of a profitable deviation. Therefore, in all equilibrium menus, the type l seller's incentive constraint binds. ■

A.1.2 Proof of Proposition 1 and Lemma 2

We prove the proposition through the following sequence of lemmas.

Lemma 1. $F_h(\cdot)$ *has no flats.*

Proof. Suppose by way of contradiction that $F_h(\cdot)$ is flat in an interval (u_{h1}, u_{h2}). In other words, there exists $(u_{l2}, u_{h2}) \in \text{Supp}(F_l) \times \text{Supp}(F_h)$ such that, for some $\bar{\varepsilon} > 0$, the distribution F_h satisfies $F_h(u_{h2}) = F_h(u_{h2} - \varepsilon)$ for all $\epsilon \in [0, \bar{\varepsilon}]$. We prove that there must exist a profitable deviation. The particular deviation we construct depends on whether $u_{l2} < u_{h2}$ or $u_{l2} = u_{h2}$ and whether F_l is flat on an interval containing u_{l2} or not. We consider each relevant case in turn:

1. Suppose that $u_{l2} < u_{h2}$. In this case, a deviation to $(u_{l2}, u_{h2} - \varepsilon')$ with $\varepsilon' < \varepsilon$ is feasible and must be profitable because such a deviation increases profits earned from trading with h types but does not change the fraction of h types attracted.

2. Suppose that $u_{l2} = u_{h2}$ and F_l is flat below u_{l2}. In this case, a deviation of the form $(u_{l2} - \varepsilon', u_{h2} - \varepsilon')$ for a small but positive ε' is profitable since it increases profits per trade (from both l and h type sellers) but does not change the fraction of either type attracted.

3. Suppose $u_{l2} = u_{h2}$ and F_l is not flat below u_{l2}. Such a situation is depicted in Figure 1. Point A represents the contract (u_{l2}, u_{h2}). Since F_h is flat by assumption, the area between the two **red** dashed lines must not contain any equilibrium menu. Since F_l is not flat below u_{l2} by assumption

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and there are no menus in the area between the red dashed lines, an equilibrium contract must exist in the region where point D is located; recall, since $u_h \geqslant u_l$, point D cannot lie below the lower red dashed line. Let point D represent such an equilibrium menu. In addition, let B represent a menu with the same offer to the low type as D but offers u_{h2} to the high type. Similarly, let C represent a menu with the same offer to the low type as A and the same offer to the high type as D.

For any distributions, F_l and F_h, the profit function, $\Pi(u_l, u_h)$, is weakly supermodular so that

$$\Pi_A + \Pi_D \leqslant \Pi_C + \Pi_B.$$

Since both D and A are offered in equilibrium, we must have that $\Pi_A = \Pi_D \geqslant \Pi_C, \Pi_B$. This implies that $\Pi_A = \Pi_B$. Additionally, since F_h is flat between B and E (and these menus offer the same u_l), it must be that $\Pi_E > \Pi_B$. Therefore, this is a profitable deviation.

■

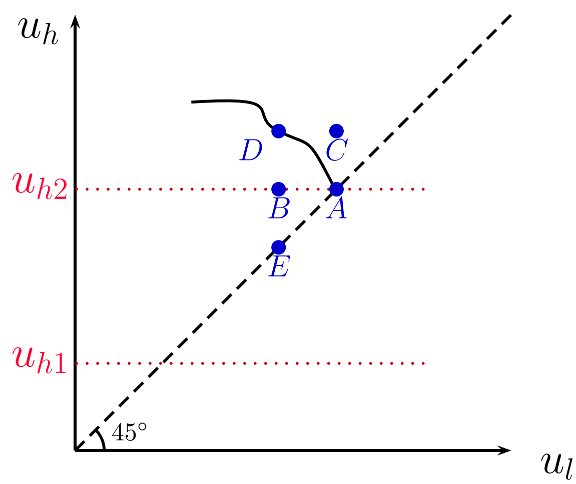

Figure 1: A graphical illustration of why F_h cannot be flat.

Lemma 2. $F_l(\cdot)$ *has no flats.*

Proof. Suppose by way of contradiction that F_l is flat in an interval (u_{l1}, u_{l2}). Without loss of generality, we can complete the measure Φ to include menus with the first element given by u_{l1} and u_{l2}. Since the profit function is weakly supermodular, then the policy correspondence must be weakly increasing. Now consider the policy correspondences $U_h(u_{l1})$ and $U_h(u_{l2})$. Note that $Cl(U_h(u_{l1}))$ and $Cl(U_h(u_{l2}))$ cannot be disjoint—if they were, then there would be a flat in the support of F_h, which contradicts Lemma 1. Let $\hat{u}_h$ be a common value in the two sets. We present a depiction of such a situation in Figure 2 below.

Holding $\hat{u}_h$ fixed, the profit function must be linear over the set (u_{l1}, u_{l2}), since $F_l(\cdot)$ is flat by assumption. Therefore, all the menus on the line AB must also deliver profits equal to equilibrium profits. However, since profits earned from trading with h types are increasing in u_l, the marginal benefit of a change in u_h is changing along the line AB. As a result, it is possible to construct an upward or downward deviation along AB that increases profits, implying the existence of a profitable deviation. ■

Lemma 3. Φ *has no mass point.*

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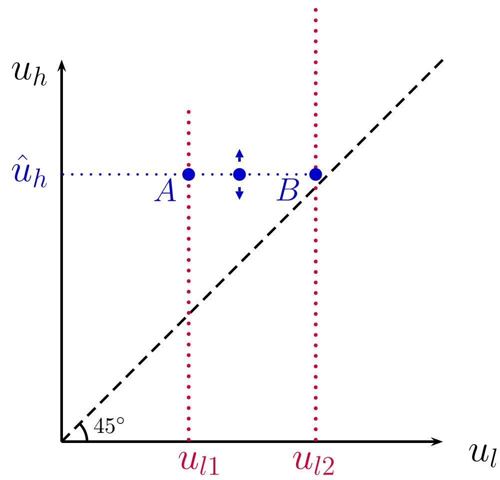

Figure 2: A graphical illustration of why F_l cannot be flat.

Proof. Suppose by way of contradiction that Φ has a mass point at the menu (u_l, u_h). Let m denote the mass at this menu. Since for any such menu, a deviation of the form $(u_l + \varepsilon_1, u_h + \varepsilon_2)$ for small $\varepsilon_1, \varepsilon_2$ (one of which is positive or negative) must be feasible, profits earned from the mass of sellers attracted to such deviation must be zero:

$$\mu_l \pi \frac{m}{2} \Pi_l(u_l) + \mu_h \pi \frac{m}{2} \Pi_h (u_l, u_h) = 0.$$

If the menu (u_l, u_h) is interior to the constraint set—that is, if $c_h - c_l > u_h - u_l > 0$—then a simple deviation along u_l or u_h will be feasible and profitable. However, it is possible that (u_l, u_h) is on the boundary of the set and, as a result, not all deviations are feasible. There are two relevant possibilities:

1. Suppose that the menu with mass, (u_l, u_h), satisfies $u_h = u_l + c_h - c_l$. In such a case, the menu features no trade with the high type. Therefore, it must be that $\Pi_h \leqslant 0$. Since equilibrium profits are strictly positive, it must be that $\Pi_l > 0$. Hence, an infinitesimal increase in u_l, which is feasible, increases profits.

2. Suppose that the menu with mass, (u_l, u_h), satisfies $u_h = u_l$. Then (u_l, u_h) is a pooling menu. Therefore, the profits from the high type must be positive. As a result, the buyer offering this contract could increase profits with an infinitesimal increase in u_h (which would attract a mass of high types) while holding u_l constant.

■

Lemma 4. $F_h(\cdot)$ *does not have a mass point.*

Proof. Suppose by way of contradiction that F_h has a mass point. From Lemma 3, we know that this mass point could not have been created from a mass point in Φ. Therefore, if F_h has a mass point at $\hat{u}_h$, it must be that a positive measure set of the form $\{(u_l, \hat{u}_h)\}$ exists. Figure 3 depicts this possibility.

Note that at one of the points on the line, profits from the h type, $\Pi_h(u_l, \hat{u}_h)$, must be nonzero since Π_h is strictly increasing in u_l. Therefore, a small deviation upward or downward increases profits; this implies the existence of a profitable deviation and yields the necessary contradiction. ■

To show that F_l has no mass points, we make use of the strict supermodularity of the profit function, which only relies on the continuity of F_h. We therefore provide a proof of the strict supermodularity of the profit function here.

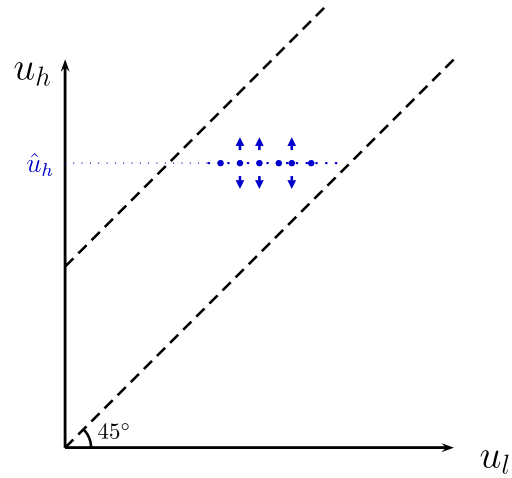

Figure 3: A graphical illustration of why F_h cannot have a mass point.

Proof of Lemma 2. Suppose $u_{l2} > u_{l1}$ and $u_{h2} > u_{h1}$. Then, letting $\xi_1 \equiv \frac{v_h - c_h}{c_h - c_l} > 0$ and $\xi_2 \equiv \frac{v_h - c_l}{c_h - c_l} > 0$,

$$\begin{aligned}
& \Pi\left(u_{l1}, u_{h2}\right) - \Pi\left(u_{l1}, u_{h1}\right) \\
= \; & \mu_h \left\{[1-\pi+\pi F_h(u_{h2})]\Pi_h(u_{l1}, u_{h2}) - [1-\pi+\pi F_h(u_{h1})]\Pi_h(u_{l1}, u_{h1})\right\} \\
= \; & \mu_h \left\{[1-\pi+\pi F_h(u_{h2})]\left[v_h + \xi_1 u_{l1} - \xi_2 u_{h2}\right] - [1-\pi+\pi F_h(u_{h1})]\left[v_h + \xi_1 u_{l1} - \xi_2 u_{h1}\right]\right\} \\
< \; & \mu_h \left\{[1-\pi+\pi F_h(u_{h2})]\left[v_h + \xi_1 u_{l2} - \xi_2 u_{h2}\right] - [1-\pi+\pi F_h(u_{h1})]\left[v_h + \xi_1 u_{l2} - \xi_2 u_{h1}\right]\right\} \\
= \; & \Pi\left(u_{l2}, u_{h2}\right) - \Pi\left(u_{l2}, u_{h1}\right),
\end{aligned}$$

where the inequality follows from the fact that F_h is strictly increasing, and hence

$$\pi\xi_1(u_{l2} - u_{l1})[F_h(u_{h2}) - F_h(u_{h1})] > 0.$$

■

Lemma 5. F_l *is continuous except possibly at* v_l.

Proof. Suppose by way of contradiction that F_l is not continuous and thus has a mass point at some $\hat{u}_l$. Again, by Lemma 3, it must be that a positive measure set of the form $S = \{(\hat{u}_l, u_h)\}$ exists. It is immediate that $\Pi_l(\hat{u}_l) = 0$; otherwise, it would be profitable to increase or decrease u_l by ε if $\Pi_l(\hat{u}_l) > 0$ or $\Pi_l(\hat{u}_l) < 0$, respectively. If $\Pi_l(\hat{u}_l) = 0$, then it must be $\hat{u}_l = v_l$. ■

A.2 Proofs from Section 4

A.2.1 Proof of Proposition 2

Proof. We first show that the equilibrium allocations constructed in (16) and (17) are indeed separating and interior. Our construction ensures that local deviations are not profitable. Below we prove that the global deviations are not profitable as well.

Verifying Allocations Are Separating and Interior. Note that the solution to the differential equation in (17), together with boundary condition $F_l\left(c_l\right) = 0$, must satisfy

$$1 - \pi + \pi F_l\left(u_l\right) = (1-\pi)\left(v_l - c_l\right)^{\phi_l}\left(v_l - u_l\right)^{-\phi_l}. \tag{1}$$

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Therefore, from (17), $U_h(u_l)$ must satisfy

$$U_h(u_l) = \frac{1}{\mu_h \frac{v_h - c_l}{c_h - c_l}} \left[\mu_h v_h + \mu_l v_l - \mu_l \phi_l u_l - \mu_l (v_l - c_l)^{1-\phi_l} (v_l - u_l)^{\phi_l} \right].$$

For the allocation to be separating, we must verify that

$$u_l + c_h - c_l \geqslant U_h(u_l) > u_l, \forall u_l \in \text{Supp}(F_l), \tag{2}$$

where

$$\text{Supp}(F_l) = \left[c_l, v_l - (1-\pi)^{\frac{1}{\phi_l}} (v_l - c_l) \right].$$

The second inequality in (2), $U_h(u_l) > u_l$, is satisfied if and only if

$$\mu_h v_h + \mu_l v_l > \mu_l (v_l - c_l)^{1-\phi_l} (v_l - u_l)^{\phi_l} + u_l \tag{3}$$

for all $u_l \in \text{Supp}(F_l)$. Let $H(u_l)$ denote the right-hand side of (3). We argue that $H(\cdot)$ is strictly concave and attains its maximum at a value $u_l^* \in [c_l, v_l]$ with $H(u_l^*) < \mu_h v_h + \mu_l v_l$, implying that (3) is satisfied for all $u_l \in \text{Supp}(F_l)$. To see this, note that

$$H'(u_l) = -\phi_l \mu_l (v_l - c_l)^{1-\phi_l} (v_l - u_l)^{\phi_l - 1} + 1 \tag{4}$$

$$H''(u_l) = \phi_l (\phi_l - 1) \mu_l (v_l - c_l)^{1-\phi_l} (v_l - u_l)^{\phi_l - 2} < 0, \tag{5}$$

where the inequality in (5) is implied by the fact that $0 < \phi_l < 1$. Also, since $\phi_l < 1$, $H'(v_l) = -\infty$ and $H'(c_l) = 1 - \phi_l \mu_l > 0$, the maximum of $H(u_l)$ is attained on the interior of $[c_l, v_l]$.

The function $H(u_l)$ is maximized at u_l^* given by

$$u_l^* = v_l - (\phi_l \mu_l)^{\frac{1}{1-\phi_l}} (v_l - c_l)$$

with

$$H(u_l^*) = v_l + (v_l - c_l)\, \mu_l^{\frac{1}{1-\phi_l}} \phi_l^{\frac{\phi_l}{1-\phi_l}} [1 - \phi_l].$$

Since $c_h \geqslant v_l$ and $\phi_l < 1$, it is immediate that

$$(\phi_l \mu_l)^{\frac{\phi_l}{1-\phi_l}} < 1 \leqslant \frac{(c_h - c_l)(v_h - v_l)}{(v_l - c_l)(v_h - c_h)},$$

which implies

$$(v_l - c_l)\, \mu_l (\phi_l \mu_l)^{\frac{\phi_l}{1-\phi_l}} \frac{\mu_h}{\mu_l} \frac{v_h - c_h}{c_h - c_l} < \mu_h (v_h - v_l).$$

Hence,

$$(v_l - c_l)\, \mu_l (\phi_l \mu_l)^{\frac{\phi_l}{1-\phi_l}} (1 - \phi_l) < \mu_h (v_h - v_l)$$

and

$$\max_{u_l \in [c_l, v_l]} H(u_l) = H(u_l^*) = v_l + (v_l - c_l)\, \mu_l (\phi_l \mu_l)^{\frac{\phi_l}{1-\phi_l}} (1 - \phi_l) < \mu_h (v_h - v_l) + v_l$$

as needed.

We now establish that the first inequality in (2) is true, which requires showing that

$$\frac{\mu_h v_h + \mu_l v_l - \mu_l \phi_l u_l - \mu_l (v_l - c_l)^{1-\phi_l} (v_l - u_l)^{\phi_l}}{\mu_h \frac{v_h - c_l}{c_h - c_l}} \leqslant u_l + c_h - c_l,$$

or, equivalently,

$$\mu_h c_l + \mu_l v_l \leqslant u_l + \mu_l (v_l - c_l)^{1-\phi_l} (v_l - u_l)^{\phi_l}, \forall u_l \in \text{Supp}(F_l) \subset [c_l, v_l]. \tag{6}$$

Since, the right side of (6) is a concave function, it takes its minimum values at the extremes of the interval $[v_l, c_l]$. These values are given by v_l and $\mu_l v_l + \mu_h c_l$, both of which are at least as large as the left side of (6). Hence, (6) must be satisfied for all $u_l \in [v_l, c_l]$, as needed.

Global Deviations. Note that our conditions (16) and (17) imply that local deviations with respect to u_h and u_l are not profitable. It, thus, remains to show that, for all (u'_l, u'_h), $\Pi(u'_l, u'_h) \leqslant \mu_l (1-\pi)(v_l - c_l)$. We consider two types of deviations:

1. Consider first deviation menus with $u'_h > \max \text{Supp}(F_h) = \bar{u}_h$. Such deviations attract all type h sellers, so that $1 - \pi + \pi F_h(u'_h) = 1$. If $u'_l > \max \text{Supp}(F_l) = \bar{u}_l$, then the profits from this menu are given by
$$\mu_l (v_l - u'_l) + \mu_h \Pi_h(u'_l, u'_h).$$
Since $\phi_l > 0$, this function is decreasing in u'_l and u'_h, and therefore
$$\mu_l (v_l - u'_l) + \mu_h \Pi_h(u'_l, u'_h) < \mu_l (v_l - \bar{u}_l) + \mu_h \Pi_h(\bar{u}_l, \bar{u}_h) = \mu_l (1-\pi)(v_l - c_l).$$
When $u'_l \leqslant \bar{u}_l$, the partial derivative of $\Pi(u'_l, u'_h)$ with respect to u'_l is
$$-\mu_l (1 - \pi + \pi F_l(u'_l)) + \mu_l \pi f_l(u'_l)(v_l - u'_l) + \mu_h \frac{v_h - c_h}{c_h - c_l} \geqslant$$
$$-\mu_l (1 - \pi + \pi F_l(u'_l)) + \mu_l \pi f_l(u'_l)(v_l - u'_l) + \mu_h (1 - \pi + \pi F_l(u'_l)) \frac{v_h - c_h}{c_h - c_l} = 0.$$
Thus, for a given value of u'_h, we must have
$$\Pi(u'_l, u'_h) \leqslant \Pi(\bar{u}_l, u'_h) < \Pi(\bar{u}_l, \bar{u}_h) = \mu_l (1-\pi)(v_l - c_l),$$
where the last inequality follows from the fact that Π_h is decreasing in u'_h. Thus, such global deviations are unprofitable.

2. Next consider deviations with $u'_h \in [c_h, \bar{u}_h]$. In this case, there must exist $\tilde{u}_l$ such that $u'_h = U_h(\tilde{u}_l)$ and thus $F_h(u'_h) = F_l(\tilde{u}_l)$. We can thus write the profits obtained from the deviation menu (u'_l, u'_h) as
$$\mu_l (1 - \pi + \pi F_l(u'_l))(v_l - u'_l) + \mu_h (1 - \pi + \pi F_l(\tilde{u}_l)) \Pi_h(u'_l, u'_h). \tag{7}$$
We show that the function defined by (7) is strictly concave in u'_l for values of $u'_l \in \text{Supp}(F_l)$ and decreasing for values of $u'_l > \bar{u}_l$ so that this function is maximized at the value of u'_l, which equates its partial derivative with zero. By (16), this partial derivative is zero when evaluated at $u'_l = \tilde{u}_l$, which completes the proof.

   Note that for $u'_l \in \text{Supp}(F_l)$, since Π_h is linear in u'_l, the second derivative of (7) with respect to u'_l is given by
$$\frac{\partial^2}{\partial (u'_l)^2} \mu_l (1 - \pi + \pi F_l(u'_l))(v_l - u'_l).$$

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Using the form of the distribution F_l given by (1), we may rewrite this second derivative as

$$\begin{aligned}\frac{\partial^2}{\partial\left(u_l'\right)^2}\mu_l\left(1-\pi+\pi F_l\left(u_l'\right)\right)\left(\nu_l-u_l'\right) &= \frac{\partial^2}{\partial\left(u_l'\right)^2}\mu_l\left(1-\pi\right)\left(\nu_l-c_l\right)^{\phi_l}\left(\nu_l-u_l'\right)^{1-\phi_l} \\ &= \left(\phi_l-1\right)\phi_l\mu_l\left(1-\pi\right)\left(\nu_l-c_l\right)^{\phi_l}\left(\nu_l-u_l'\right)^{-1-\phi_l}<0\end{aligned}$$

so that (7) is strictly concave in u_l' for values of $u_l' \in \text{Supp}\left(F_l\right)$. For values $u_l' > \tilde{u}_l$, $1-\pi+\pi F_l\left(u_l'\right)=1$, and thus (7) satisfies

$$\mu_l\left(\nu_l-u_l'\right)+\mu_h\left(1-\pi+\pi F_l\left(\tilde{u}_l\right)\right)\Pi_h\left(u_l', u_h'\right).$$

The derivative of this function with respect to u_l' is given by

$$-\mu_l+\mu_h\left(1-\pi+\pi F_l\left(\tilde{u}_l\right)\right)\frac{\nu_h-c_h}{c_h-c_l}<-\mu_l+\mu_h\frac{\nu_h-c_h}{c_h-c_l}=-\mu_l\phi_l<0.$$

Therefore, (7) is maximized at a value of u_l', which equates the partial derivative of (7) with zero. This value must satisfy

$$-\mu_l\left(1-\pi+\pi F_l\left(u_l'\right)\right)+\mu_l\pi f_l\left(u_l'\right)\left(\nu_l-u_l'\right)+\mu_h\left(1-\pi+\pi F_l\left(\tilde{u}_l\right)\right)\frac{\nu_h-c_h}{c_h-c_l}=0.$$

Note that since (7) is strictly concave, at most one u_l' exists that satisfies the above. The differential equation (16) implies that $u_l'=\tilde{u}_l$ is a solution to the above equation. This implies that (7) must be maximized at $u_l'=\tilde{u}_l$.

A.2.2 Proofs of Propositions 3 and 4

We prove these propositions together. To begin, let ϕ_1 be the value of ϕ_l that satisfies

$$c_h \geqslant \nu_l+\frac{\pi(1-\mu_l)\left(\nu_h-\nu_l\right)}{(1-\pi)\left[(1-\pi)^{\frac{1-\phi_l}{\phi_l}}-1\right]} \tag{8}$$

with equality. Similarly, let ϕ_2 be the value of ϕ_l that satisfies

$$1-\pi \geqslant \frac{\mu_h\nu_h+\mu_l\nu_l-\nu_l}{(1-\phi_l)(\mu_h\nu_h+\mu_l\nu_l-c_h)} \tag{9}$$

with equality. We first argue that (8) represents a lower bound on ϕ_l and (9) represents an upper bound on ϕ_l, which lies below the lower bound defined by (8). In other words, the inequalities (8) and (9) partition the set $(-\infty, 0]$. We then prove that the equilibrium described in Proposition 4 exists—that is, in each case, no profitable local or global deviations exist when buyers use the equilibrium strategies defined jointly by Propositions 3 and 4.

Lemma 6. *(8) is satisfied if and only if $\phi_1 \leqslant \phi_l < 0$ and (9) is satisfied if and only if $\phi_l \leqslant \phi_2$. Moreover, $\phi_2 < \phi_1 < 0$.*

Proof. First, note that equation (8), which implicitly determines the threshold ϕ_1, can be rewritten as

$$(1-\pi)^{\frac{1-\phi_l}{\phi_l}} \geqslant \frac{\pi}{1-\pi}\frac{\nu_h-\nu_l}{c_h-\nu_l}\mu_h+1, \tag{10}$$

or, after taking logs and substituting for ϕ_l, can be rewritten as

$$\frac{\mu_h(\nu_h - c_h)}{c_h - c_l - \mu_h(\nu_h - c_l)} \log(1-\pi) - \log(\mu_h \pi(\nu_h - \nu_l) + (1-\pi)(c_h - \nu_l)) - \log[(1-\pi)(c_h - \nu_l)] \geqslant 0. \tag{11}$$

We show that the left side of (11) is a decreasing function of μ_h, that (11) is strictly satisfied when μ_h is such that $\phi_l = 0$, and that (11) is weakly violated when $\mu_h = 1$. Hence, there is a unique threshold μ_1 (and implied threshold ϕ_1) such that for all $\mu_h \leqslant \mu_1$ such that $\phi_l < 0$, the separating condition (8) is satisfied. Differentiating the left side of (11) with respect to μ_h, we obtain

$$\log(1-\pi)\frac{(\nu_h - c_h)(c_h - c_l)}{[c_h - c_l - \mu_h(\nu_h - c_l)]^2} - \frac{\pi(\nu_h - \nu_l)}{\mu_h \pi(\nu_h - \nu_l) + (1-\pi)(c_h - \nu_l)},$$

which is negative for all $\pi \leqslant 1$. Next, as $\phi_l \to 0$ from below, it is immediate that (10) is satisfied since the left-hand side tends to infinity. As $\mu_h \to 1$, the term $(1-\phi_l)/\phi_l \to -1$ and so (10) tends to the requirement that

$$1 \geqslant \pi \frac{\nu_h - \nu_l}{c_h - \nu_l} + (1-\pi),$$

which is violated since $c_h < \nu_h$.

Next, consider equation (9), which implicitly determines the threshold ϕ_2. Substituting for ϕ_l, one can show the inequality (9) is equivalent to

$$\mu_h(\nu_h - \nu_l)\left[1 + (1-\pi)\frac{\nu_h - c_h}{c_h - c_l}\right] \geqslant \nu_h - \nu_l + (c_h - \nu_l)(1-\pi)\frac{\nu_h - c_h}{c_h - c_l}. \tag{12}$$

Clearly, (12) represents a lower bound on μ_h, or, equivalently, an upper bound on ϕ_l. Note that this equation is necessarily satisfied at $\mu_h = 1$. It is immediate that when μ_h is such that $\phi_l = 0$, equation (9) is violated since $c_h > \nu_l$.

We now establish that $\phi_2 < \phi_1$ by proving that $\phi_l \leqslant \phi_2$ implies $\phi_l < \phi_1$. Suppose $\phi_l \leqslant \phi_2$ and let $\bar{\nu} = \mu_h \nu_h + \mu_l \nu_l$, so that we can write (9) as

$$1 - \pi \geqslant \frac{\bar{\nu} - \nu_l}{(1-\phi_l)(\bar{\nu} - c_h)}. \tag{13}$$

Below, we will use the fact that (13) implies

$$1 - \phi_l \geqslant \frac{\bar{\nu} - \nu_l}{(\bar{\nu} - c_h)(1-\pi)} > \frac{\bar{\nu} - \nu_l}{\bar{\nu} - c_h} \quad \Rightarrow \quad -\phi_l > \frac{c_h - \nu_l}{\bar{\nu} - c_h}.$$

To prove that (8) is violated when $\phi_l \leqslant \phi_2$, note that (8) can be rearranged as

$$(1-\pi)\left[(1-\pi)^{\frac{1-\phi_l}{\phi_l}} - 1\right](c_h - \nu_l) - \pi\mu_h(\nu_h - \nu_l) \geqslant 0$$

which can be simplified to

$$(1-\pi)(\bar{\nu} - c_h) + (1-\pi)^{\frac{1}{\phi_l}}(c_h - \nu_l) \geqslant \bar{\nu} - \nu_l. \tag{14}$$

We will show that (14) is violated if (13) holds. Towards this end, define a function

$$H(\pi) = (1-\pi)(\bar{\nu} - c_h) + (1-\pi)^{\frac{1}{\phi_l}}(c_h - \nu_l)$$

so that we must show $H(\pi) < \bar{\nu} - \nu_l$. We argue that $H(\cdot)$ is a strictly convex function that is decreasing

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at $\pi = 0$ and that, if π satisfies (13), then $H(\pi) < H(0) = \bar{v} - v_l$, which completes the proof.

First, note that $H(\cdot)$ is strictly convex, since $\phi_l < 0$, and

$$H'(\pi) = -(\bar{v} - c_h) - \frac{1}{\phi_l}(1-\pi)^{\frac{1}{\phi_l}-1}(c_h - v_l),$$
$$H''(\pi) = \frac{1}{\phi_l}\left(\frac{1}{\phi_l} - 1\right)(1-\pi)^{\frac{1}{\phi_l}-2}(c_h - v_l) > 0.$$

Next, observe that $H(0) = \bar{v} - v_l$, $H'(0) \leqslant 0$ when $-\phi_l \geqslant (c_h - v_l)/(\bar{v} - c_h)$ and $\lim_{\pi \to 1} H(\pi) = \infty$. Thus, there is a unique value $\pi^s > 0$ such that for all $\pi < \pi^s$, $H(\pi) \leqslant \bar{v} - v_l$.

Next, let $\hat{\pi}$ denote the value of π such that (13) is satisfied with equality. We will prove that $H(\hat{\pi}) < \bar{v} - v_l$, so that $H(\pi) < \bar{v} - v_l$ for all $\pi \leqslant \hat{\pi}$.

Using the expression for $H(\pi)$, we have

$$H(\hat{\pi}) = \frac{\bar{v} - v_l}{(1-\phi_l)(\bar{v} - c_h)}(\bar{v} - c_h) + \left(\frac{\bar{v} - v_l}{(1-\phi_l)(\bar{v} - c_h)}\right)^{\frac{1}{\phi_l}}(c_h - v_l). \tag{15}$$

Straightforward algebra can be applied to (15) to show that $H(\hat{\pi}) < \bar{v} - v_l$ if and only if

$$\left(\frac{c_h - v_l}{\bar{v} - c_h}\right)^{\phi_l}\left(\frac{\bar{v} - v_l}{\bar{v} - c_h}\right)^{1-\phi_l} > (-\phi_l)^{\phi_l}(1-\phi_l)^{1-\phi_l}. \tag{16}$$

Since $(\bar{v} - v_l)/(\bar{v} - c_h) = 1 + (c_h - v_l)/(\bar{v} - c_h)$, if we let $B(x) = x^{\phi_l}(1+x)^{1-\phi_l}$, then (16) can be written as the requirement that

$$B\left(\frac{c_h - v_l}{\bar{v} - c_h}\right) > B(-\phi_l).$$

It is straightforward to show that $B'(x) < 0$ when $0 < x < -\phi_l$, and since (13) implies $-\phi_l > (c_h - v_l)/(\bar{v} - c_h)$, (16) must hold. Consequently, $H(\pi) < H(\hat{\pi}) < \bar{v} - v_l$, which proves that $\phi_1 > \phi_2$.

Definition of the Threshold, $\hat{u}_l$. To prove Propositions 3 and 4, we first define the threshold $\hat{u}_l$ for various values of $\phi_l < 0$.

Case 1: $\phi_l \leqslant \phi_2$. The threshold satisfies $\hat{u}_l = \bar{u}_l$, the upper bound of F_l, where $\bar{u}_l$ satisfies

$$\bar{v} - \bar{u}_l = (1-\pi)(\bar{v} - c_h). \tag{17}$$

Case 2: $\phi_2 < \phi_l < \phi_1$. The threshold satisfies

$$v_l + (\hat{u}_l - v_l)\left[1 - \pi + \pi F_l(\hat{u}_l)\right]^{\frac{1}{\phi_l}} = \bar{v} - (1-\pi)(\bar{v} - c_h) \tag{18}$$

where $F_l(\hat{u}_l)$ satisfies (18). As we will see below, in this case, the threshold will be such that $F_l(\hat{u}_l) \in (0,1)$ so that the equilibrium is indeed mixed.

Case 3: $\phi_1 < \phi_l < 0$. The threshold is any value such that $\hat{u}_l < \underline{u}_l$ where the lower bound of the support of F_l satisfies

$$(1-\pi)\left[\mu_l(v_l - \underline{u}_l) + \mu_h \Pi_h(\underline{u}_l, c_h)\right] = \bar{v} - \left[v_l + (1-\pi)^{\frac{1}{\phi_l}}(\underline{u}_l - v_l)\right]. \tag{19}$$

This equation determines the lower bound as the value that equates profits from the worst (separating) menu and the best (pooling) menu, where the best menu is determined as the value of u_l such that $F_l(u_l) = 1$ when F_l is determined by (16).

We now prove that the conjectured equilibria defined implicitly by the thresholds ϕ_1 and ϕ_2 are indeed equilibria.

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Lemma 7. *Suppose $\phi_1 \leqslant \phi_l < 0$. There exists an equilibrium with only separating menus.*

Proof. It suffices to ensure that global deviations are unprofitable for buyers since, by construction, the distribution $F_l(u_l)$ ensures no local deviations are profitable. To rule out global deviations, a proof similar to that of Proposition 2 can be used. We show that for a given value of u'_h, the profit function is strictly concave in u'_l and, therefore, it must be maximized at $u'_l = U_h^{-1}(u'_h)$, since at this value the derivative of the profit function is equal to zero (by construction).

Profits from such a global deviation are given by

$$\mu_l \left(1-\pi+\pi F_l\left(u'_l\right)\right)\left(\nu_l - u'_l\right) + \mu_h\left(1-\pi+\pi F_h\left(u'_h\right)\right)\Pi_h\left(u'_l, u'_h\right).$$

Since Π_h is linear in u'_l, the second derivative of the above function is equal to the second derivative of profits from l type sellers. Using (16), we know that $\left(1-\pi+\pi F_l\left(u'_l\right)\right) = \kappa\left(u'_l - \nu_l\right)^{-\phi_l}$ for some non-negative constant κ. Therefore, we have

$$\begin{aligned}\frac{\partial^2}{\partial\left(u'_l\right)^2}\mu_l\left(1-\pi+\pi F_l\left(u'_l\right)\right)\left(\nu_l - u'_l\right) &= -\mu_l\kappa\frac{\partial^2}{\partial\left(u'_l\right)^2}\left(u'_l-\nu_l\right)^{1-\phi_l} \\ &= -\mu_l\kappa\left(1-\phi_l\right)\left(-\phi_l\right)\left(u'_l-\nu_l\right)^{-1-\phi_l} < 0.\end{aligned}$$

Lemma 8. *Suppose $\phi_l \leqslant \phi_2$. There exists an equilibrium with only pooling menus.*

Proof. We first prove that no local deviations in the pooling equilibrium strictly improve profits. Below we demonstrate global deviations are also unprofitable. Recall that in an equilibrium with only pooling menus, the distribution $F_l\left(u_l\right)$ satisfies

$$\left(1-\pi+\pi F_l(u_l)\right)\left(\bar{\nu}-u_l\right) = \left(1-\pi\right)\left(\bar{\nu}-c_h\right), \tag{20}$$

where $\bar{\nu} = \mu_h\nu_h + \mu_l\nu_l$, $U_h(u_l) = u_l$, $F_h(u_l) = F_l(u_l)$, and $\mathrm{Supp}(F_l) = \left[c_h, \bar{\nu} - (1-\pi)\left(\bar{\nu}-c_h\right)\right]$. Fix any utility, u_l, interior to the support of F_l and consider a local deviation to the menu $\left(u'_l, u'_h\right) = \left(u_l, u_l + \varepsilon\right)$. Profits from such a local deviation satisfy

$$\begin{aligned}&\mu_l\left(1-\pi+\pi F_l(u_l)\right)\left(\nu_l-u_l\right) + \mu_h\left(1-\pi+\pi F_l\left(u_l+\varepsilon\right)\right)\Pi_h\left(u_l, u_l+\varepsilon\right) \\ =\;& \mu_l\left(1-\pi+\pi F_l(u_l)\right)\left(\nu_l-u_l\right) + \mu_h\left(1-\pi+\pi F_l\left(u_l+\varepsilon\right)\right)\left[\nu_h - u_l - \varepsilon\frac{\nu_h-c_l}{c_h-c_l}\right].\end{aligned}$$

If local deviations are unprofitable, this function must be maximized at $\varepsilon = 0$, so that F_l must satisfy

$$\mu_h\pi f_l\left(u_l\right)\left[\nu_h-u_l\right] - \mu_h\left(1-\pi+\pi F_l\left(u_l\right)\right)\frac{\nu_h-c_l}{c_h-c_l} \leqslant 0.$$

Totally differentiating (20) yields the following relationship between F_l and f_l,

$$\pi f_l(u_l)\left(\bar{\nu}-u_l\right) = \left(1-\pi+\pi F_l(u_l)\right) \tag{21}$$

so that local deviations are unprofitable if

$$\mu_h\pi f_l\left(u_l\right)\left[\nu_h-u_l\right] - \mu_h\pi f_l(u_l)\left(\bar{\nu}-u_l\right)\frac{\nu_h-c_l}{c_h-c_l} \leqslant 0.$$

Since F_l is continuous in our constructed equilibrium, we may simplify this condition using straightforward algebra as

$$u_l\left(\nu_h-c_h\right) \leqslant \bar{\nu}\left(\nu_h-c_l\right) - \nu_h\left(c_h-c_l\right).$$

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Consequently, we see that it suffices to check that this deviation is unprofitable at $\max \text{Supp}(F_l)$. Using $u_l = \bar{v} - (1-\pi)(\bar{v} - c_h)$, simple algebraic manipulations show that this local deviation is unprofitable as long as

$$\frac{\bar{v} - v_l}{(1-\phi_l)(\bar{v} - c_h)} \leqslant 1 - \pi, \tag{22}$$

which is guaranteed by Lemma 6 since $\phi_l \leqslant \phi_2$.

To rule out global deviations, we show that for any value of $u'_h \in \text{Supp}(F_l)$, the profit function is increasing in u'_l for all $u'_l \leqslant u'_h$. Thus, profits are maximized at the pooling menu $u'_l = u'_h$ so that there are no profitable deviations.

Profits associated with any global deviation (u'_l, u'_h) with $u'_l \leqslant u'_h$ and $u'_h \in \text{Supp}(F_l)$ are given by

$$\mu_l \left(1 - \pi + \pi F_l(u'_l)\right)\left(v_l - u'_l\right) + \mu_h \left(1 - \pi + \pi F_l\left(u'_h\right)\right) \Pi_h \left(u'_l, u'_h\right).$$

Differentiating, we obtain

$$\begin{aligned}
&\mu_l \pi f_l\left(u'_l\right)\left(v_l - u'_l\right) - \mu_l\left(1 - \pi + \pi F_l(u'_l)\right) + \mu_h\left(1 - \pi + \pi F_l\left(u'_h\right)\right)\frac{v_h - c_h}{c_h - c_l} \geqslant \\
&\mu_l \pi f_l\left(u'_l\right)\left(v_l - u'_l\right) - \mu_l\left(1 - \pi + \pi F_l(u'_l)\right) + \mu_h\left(1 - \pi + \pi F_l\left(u'_l\right)\right)\frac{v_h - c_h}{c_h - c_l} = \\
&\mu_l \pi f_l\left(u'_l\right)\left(v_l - u'_l\right) - \mu_l \phi_l\left(1 - \pi + \pi F_l(u'_l)\right)
\end{aligned} \tag{23}$$

where the inequality follows from the fact that $u'_l \leqslant u'_h$ so that $F_l(u'_h) \geqslant F_l(u'_l)$. Using (21) to substitute for $\pi f_l(u'_l)$, we can write the last line of (23) as

$$\mu_l(1 - \pi + \pi F_l(u'_l))\left[1 + \frac{v_l - \bar{v}}{\bar{v} - u'_l} - \phi_l\right].$$

Since $u'_l \leqslant u'_h \leqslant \max \text{Supp}(F_l)$, the expression in brackets takes its minimum value at $u'_l = \max \text{Supp}(F_l)$ so that

$$1 + \frac{v_l - \bar{v}}{\bar{v} - u'_l} - \phi_l \geqslant 1 + \frac{v_l - \bar{v}}{(1-\pi)(\bar{v} - c_h)} - \phi_l \geqslant 0,$$

where the second inequality follows from (22). This implies that the expression in (23) is positive so that profits are globally maximized at $u'_l = u'_h$ for all $u'_h \in \text{Supp}(F_l)$. ∎

Lemma 9. *Suppose $\phi_2 < \phi_l < \phi_1$. There exists a mixed equilibrium.*

Proof. Recall that the threshold $\hat{u}_l$ is such that the constructed equilibrium features pooling contracts for $u_l \in [\min \text{Supp}(F_l), \hat{u}_l]$ and separating menus for $u_l \in (\hat{u}_l, \max \text{Supp}(F_l))$. First, we claim that when $\phi_2 < \phi_l < \phi_1$, then $\hat{u}_l$ is interior, in the sense that $c_h < \hat{u}_l < \bar{u}(\hat{u}_l)$. Second, we prove that no local or global deviations are profitable.

To see that $\hat{u}_l$ is interior, conjecture that $\hat{u}_l > c_h$ (we will verify it later), in which case $\hat{u}_l$ must satisfy[1]

$$\bar{v} - \left\{v_l + (\hat{u}_l - v_l)\left[(1-\pi)\frac{\bar{v} - c_h}{\bar{v} - \hat{u}_l}\right]^{\frac{1}{\phi_l}}\right\} - (1-\pi)(\bar{v} - c_h) = 0. \tag{24}$$

Let $H(\hat{u}_l)$ denote the left-hand side of (24). We will prove that when $\phi_2 < \phi_l < \phi_1$, there are two solutions to $H(\hat{u}_l) = 0$ with $\hat{u}_l > c_h$.

First, observe that one solution to $H(\hat{u}_l) = 0$ is given by

$$\hat{u}_l = \bar{u} = \bar{v} - (1-\pi)(\bar{v} - c_h).$$

[1]Recall that equilibrium profits satisfy $\bar{\Pi} = (1-\pi)(\bar{v} - c_h)$ when the worst menu offered in equilibrium is the pooling, monopsony menu.

This solution coincides with the conjecture that all menus are pooling, and therefore $\bar{u}(\hat{u}_l) = \hat{u}_l$.

We argue that there exists another solution $\hat{u}_l \in (c_h, \bar{u})$. We show this by proving that $H(\cdot)$ is convex, $H(c_h) > 0$, and $H'(\bar{u}) > 0$ so that an additional solution in the interval $(c_h, \bar{u})$ must exist.

Note that

$$H'(u) = -\left[(1-\pi)\frac{\bar{v}-c_h}{\bar{v}-u}\right]^{\frac{1}{\phi_l}} - (u - v_l)\frac{1}{\phi_l}\left[(1-\pi)\frac{\bar{v}-c_h}{\bar{v}-u}\right]^{\frac{1}{\phi_l}-1}(1-\pi)(\bar{v}-c_h)(\bar{v}-u)^{-2}.$$

By differentiating $H'(\cdot)$ and applying extensive algebraic manipulations (available upon request), one can show that $H''(\cdot) \geqslant 0$. Recall that $\bar{u}$ is defined so that $H(\bar{u}) = 0$ and

$$H'(\bar{u}) = -1 - \frac{1}{\phi_l}\frac{\bar{u}-v_l}{\bar{v}-\bar{u}} = H'(\bar{u}) = \frac{1-\phi_l}{\phi_l}\left[1 - \frac{\bar{v}-v_l}{(1-\pi)(1-\phi_l)(\bar{v}-c_h)}\right],$$

where the second equality is obtained by substituting for $\bar{u}$ and rearranging terms. When $\phi_l > \phi_2$, the term in brackets is negative, by Lemma 6, so that $H'(\bar{u}) > 0$. Finally, one can show that $H(c_h)$ satisfies

$$H(c_h) = \frac{1}{(1-\pi)^{\frac{1}{\phi_l}} - (1-\pi)}\left[v_l + \frac{\pi(\bar{v}-v_l)}{(1-\pi)^{\frac{1}{\phi_l}} - (1-\pi)} - c_h\right].$$

From Lemma 6, since $\phi_l < \phi_1 < 0$, the term in brackets is strictly positive, and, since the leading fraction is also positive, we must have $H(c_h) > 0$.

Hence, when $\phi_2 < \phi_l < \phi_1 < 0$, there must exist a solution to $H(\hat{u}_l) = 0$ with $\hat{u}_l \in (c_h, \bar{u})$. When $\hat{u}_l < \bar{u}$, one can show that $F_l(\hat{u}_l) < 1$ when F_l is determined by (18) on the interval $[c_h, \hat{u}_l]$, which confirms the conjecture that $\hat{u}_l$ is the interior of the support of F_l.

We now show that buyers cannot improve their profits by deviating from the constructed mixed allocation. As in Lemma 7 with only separation, the distribution F_l for $u_l \in [\hat{u}_l, \max \text{Supp}(F_l)]$ is chosen to ensure local deviations are not profitable. It remains to show, then, that local deviations are not profitable in the pooling region and that no global deviations are profitable. As in Lemma 8 with only pooling menus, it suffices to ensure that at the upper bound of the pooling region, $\hat{u}_l$, no local deviations are profitable, or

$$\hat{u}_l(v_h - c_h) \leqslant \bar{v}(v_h - c_l) - v_h(c_h - c_l). \tag{25}$$

To prove that (25) holds, first note that since $\phi_2 < \phi_l < \phi_1$, we have $c_h < \hat{u}_l < \bar{u}(\hat{u}_l)$. We now prove that (25) is satisfied at $\hat{u}_l$. Algebra (available upon request) shows that (25) may be written as

$$\hat{u}_l \leqslant \frac{-\phi_l}{1-\phi_l}\bar{v} + \frac{1}{1-\phi_l}v_l.$$

Since $H(\hat{u}_l) = 0$, if $H\left(\frac{-\phi_l}{1-\phi_l}\bar{v} + \frac{1}{1-\phi_l}v_l\right) \leqslant 0$ then since $H(\cdot)$ is convex, (25) must be satisfied.

Using the form of $H(\cdot)$ implied by the left-hand side of (24), one can show that

$$H\left(\frac{-\phi_l}{1-\phi_l}\bar{v} + \frac{1}{1-\phi_l}v_l\right) = (\bar{v}-v_l)\left[\frac{\bar{v}-v_l-(1-\pi)(\bar{v}-c_h)}{\bar{v}-v_l} + \phi_l\frac{(1-\phi_l)^{\frac{1}{\phi_l}-1}(1-\pi)^{\frac{1}{\phi_l}}(\bar{v}-c_h)^{\frac{1}{\phi_l}}}{(\bar{v}-v_l)^{\frac{1}{\phi_l}}}\right]. \tag{26}$$

We now show that the term in brackets on the right side of (26) is negative. To simplify notation, define $\xi = (1-\pi)(\bar{v}-c_h)/(\bar{v}-v_l)$ so that the term in brackets can be written compactly as

$$1 - \xi + \phi_l(1-\phi_l)^{\frac{1}{\phi_l}-1}\xi^{\frac{1}{\phi_l}}.$$

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Let $G(\xi) = 1 - \xi + \phi_l (1-\phi_l)^{\frac{1}{\phi_l}-1} \xi^{\frac{1}{\phi_l}}$ and observe that for $\xi \leqslant 1/(1-\phi_l)$, we have

$$G'(\xi) = -1 + [(1-\phi_l)\,\xi]^{\frac{1}{\phi_l}-1} \geqslant 0$$

so that for low values of ξ, $G(\xi)$ is an increasing function.

Since $\phi_l > \phi_2$, (13) implies that $\xi < 1/(1-\phi_l)$. Moreover, since $G(1/(1-\phi_l)) = 0$, it must be that $G(\xi) \leqslant G(1/(1-\phi_l)) \leqslant 0$, which ensures the term in brackets in (26) is indeed negative as desired.

To rule out global deviations, one can use the arguments provided in the proofs of Lemmas 7 and 8 in each region of the Supp (F_l). Since the arguments are exact replicas of the arguments above, we omit them here. ■

A.2.3 Proof of Theorem 2

We begin with a lemma that ensures the marginal distribution F_l is continuous (i.e., it has no mass points) when $\phi_l \neq 0$. We then prove uniqueness of the equilibrium first for $\phi_l > 0$ and then for $\phi_l < 0$. (In Appendix D, we demonstrate uniqueness for $\phi_l = 0$.)

Lemma 10. *If $\phi_l \neq 0$, then F_l is continuous.*

Proof. Recall from Lemma 5 that if F_l has a mass point, then it occurs at $\hat{u}_l = v_l$. As well, from Lemma 3, there must exist a positive measure set $S = \{\hat{u}_l, u_h)\}$ such that each equilibrium menu $(\hat{u}_l, u_h)$ has $\Pi_l = 0$. Let $\underline{u}_h$ denote the lowest value of u_h for which $(\hat{u}_l, u_h)$ belongs to the closure of the set S and let $\bar{u}_h$ denote the highest such value. Without loss of generality, we may assume that $(\hat{u}_l, \bar{u}_h)$ and $(\hat{u}_l, \underline{u}_h)$ belong to S and thus deliver the same profits to a buyer as the equilibrium level of profits.

Consider then the value of two different deviations, $(\hat{u}_l - \varepsilon, \underline{u}_h)$ and $(\hat{u}_l + \varepsilon, \bar{u}_h)$, for a small value of $\varepsilon > 0$, both of which must be feasible. The profits from these deviations are given by

$$\begin{aligned}
\Pi(\hat{u}_l - \varepsilon, \underline{u}_h) &= \mu_h (1-\pi+\pi F_h(\underline{u}_h))\, \Pi_h(\hat{u}_l - \varepsilon, \underline{u}_h) + \mu_l (1-\pi+\pi F_l(\hat{u}_l - \varepsilon))\,\varepsilon \\
\Pi(\hat{u}_l + \varepsilon, \bar{u}_h) &= \mu_h (1-\pi+\pi F_h(\bar{u}_h))\, \Pi_h(\hat{u}_l + \varepsilon, \bar{u}_h) - \mu_l (1-\pi+\pi F_l(\hat{u}_l + \varepsilon))\,\varepsilon.
\end{aligned}$$

These equalities are valid because F_h does not have a mass point and F_l does not have a mass point for $u_l > v_l$ or $u_l < v_l$. Since F_l is then left or right differentiable at $\hat{u}_l$, we have that

$$\begin{aligned}
\left.\frac{d}{d\varepsilon}\Pi(\hat{u}_l - \varepsilon, \underline{u}_h)\right|_{\varepsilon=0} &= -\mu_h (1-\pi+\pi F_h(\underline{u}_h))\frac{v_h - c_h}{c_h - c_l} + \mu_l \left(1-\pi+\pi F_l^-(\hat{u}_l)\right) \\
\left.\frac{d}{d\varepsilon}\Pi(\hat{u}_l + \varepsilon, \bar{u}_h)\right|_{\varepsilon=0} &= \mu_h (1-\pi+\pi F_h(\bar{u}_h))\frac{v_h - c_h}{c_h - c_l} - \mu_l \left(1-\pi+\pi F_l^+(\hat{u}_l)\right).
\end{aligned}$$

The optimality of menus in S implies that both of these expressions must be non-positive. Since the equilibrium distributions are well-behaved above and below v_l, the equilibrium necessarily exhibits the strict rank-preserving property by Theorem 1 and therefore, $F_l^-(\hat{u}_l) = F_h(\underline{u}_h)$ and $F_l^+(\hat{u}_l) = F_h(\bar{u}_h)$. As a result, the above inequalities imply that

$$\begin{aligned}
-\mu_h \frac{v_h - c_h}{c_h - c_l} + \mu_l &\leqslant 0 \\
\mu_h \frac{v_h - c_h}{c_h - c_l} - \mu_l &\leqslant 0.
\end{aligned}$$

When $\phi_l \neq 0$, one of these is violated. Hence, a profitable deviation exists yielding the necessary contradiction. ■

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Case 1: $\phi_l > 0$. As we have shown, any separating equilibrium is uniquely determined. Thus, in order to show the uniqueness of the equilibrium in this case, it remains to show that any equilibrium is separating. To see this, suppose to the contrary that $u_l = u_h$ for some menu offered in equilibrium. Now, consider the following alternative menu $(u_l - \epsilon, u_h)$ for a small and positive value of ϵ. This menu is feasible and the change in the profits for a small value of ϵ is given by

$$\mu_l(1-\pi+\pi F_l(u_l))\epsilon - \mu_h(1-\pi+\pi F_h(u_h))\frac{v_h - c_h}{c_h - c_l}\epsilon - \mu_l \pi f_l^-(u_l)(v_l - u_l)\epsilon,$$

where f_l^- is the left derivative of F_l at u_l; recall from Appendix A.1.2 that F_l must be differentiable.

Using the definition of ϕ_l and the strict rank-preserving property, we can write the above as

$$\mu_l \phi_l (1-\pi+\pi F_l(u_l))\epsilon - \mu_l \pi f_l^-(u_l)(v_l - u_l)\epsilon.$$

The above expression must be positive: $\phi_l > 0$, F_l and $f_l^-(u_l)$ are weakly positive, and $u_l > v_l$ since $u_l = u_h \geqslant c_h > v_l$ where $c_h > v_l$ by the lemons assumption. Therefore, this alternative menu is a profitable deviation that yields the necessary contradiction.

Case 2: $\phi_l < 0$. To prove the equilibrium characterized in Proposition 4 is unique, we first prove that in any equilibrium with $\phi_l < 0$, if $\bar{u} = \max \text{Supp}(F_l)$, then $U_h(\bar{u}) = \bar{u}$ so that the best menu in equilibrium is a pooling menu. Next, we prove that if the equilibrium has a pooling region, the region begins at the lower bound of the support of F_l or ends at the upper bound of F_l. Additionally, if the equilibrium features a separating region, this region must end at the upper bound of the support of F_l. These results imply that any equilibrium must take one of the three forms described in Proposition 4: only separating, only pooling, or mixed. Finally, we show that the necessary conditions for each type of equilibrium to exist are mutually exclusive so that at most one type of equilibrium exists for each region of the parameter space, ensuring our equilibrium is unique for all $\phi_l < 0$. We prove these results in the following sequence of lemmas.

Lemma 11. *If $\phi_l < 0$, then the best equilibrium menu is a pooling menu.*

Proof. Let $\bar{u} = \max \text{Supp}(F_l)$ and suppose for contradiction that $U_h(\bar{u}) > \bar{u}$. Consider a deviation menu with $\left(u_l', u_h'\right) = (\bar{u} + \varepsilon, U_h(\bar{u}))$. Since $U_h(\bar{u}) > \bar{u}$, this menu is incentive compatible and has $F_l(u_l') = F_l\left(u_h'\right) = 1$. This menu increases the buyer's profits relative to the menu $(\bar{u}, U_h(\bar{u}))$ by the amount

$$-\mu_l \varepsilon + \mu_h \frac{v_h - c_h}{c_h - c_l} = -\mu_l \phi_l \varepsilon > 0,$$

where the inequality follows from $\phi_l < 0$. This profitable deviation yields the necessary contradiction so that we must have $U_h(\bar{u}) = \bar{u}$. ■

Lemma 12. *If $\phi_l < 0$ and an equilibrium features $[u_1, u_2] \subseteq \text{Supp}(F_l)$ such that $U_h(u_l) = u_l$ for $u_l \in [u_1, u_2]$, then either $u_1 = \min \text{Supp}(F_l)$ or $u_2 = \max \text{Supp}(F_l)$.*

Proof. Suppose toward a contradiction that a pooling interval with $u_1 > \min \text{Supp}(F_l)$ and $u_2 < \max \text{Supp}(F_l)$ exists. Then there must exist intervals sufficiently close to and below u_1 and above u_2, respectively, in which the equilibrium menus feature separation. Since in these intervals, $U_h(u_l) > u_l$ but $U_h(u_1) = u_1$ and $U_h(u_2) = u_2$, we must have $\lim_{u_l \nearrow u_1} U_h'(u_l) \leqslant 1$ and $\lim_{u_l \searrow u_2} U_h'(u_l) \geqslant 1$. In any region with $U_h(u_l) > u_l$, the distribution F_l must also satisfy

$$\frac{\pi f_l(u_l)}{1-\pi+\pi F_l(u_l)} = \frac{-\phi_l}{u_l - v_l},$$

since local deviations must be unprofitable. Moreover, in any such region, by the equal profit condition, U_h must satisfy

$$\bar{v} - \mu_l \phi_l u_l - \mu_h \frac{v_h - c_l}{c_h - c_l} U_h(u_l) = \bar{\Pi} \left(1 - \pi + \pi F_l(u_l)\right)^{-1},$$

where $\bar{\Pi}$ denotes the level of equilibrium profits.

Using these features of the conjectured equilibrium, in the separating regions, $U_h'(u_l)$ satisfies

$$-\mu_l \phi_l - (1 - \mu_l \phi_l) U_h'(u_l) = \frac{\bar{\Pi}}{1 - \pi + \pi F_l(u_l)} \frac{\phi_l}{u_l - v_l}$$

and so U_h'' satisfies

$$-(1 - \mu_l \phi_l) U_h''(u_l) = \frac{\bar{\Pi} \pi f_l(u_l)}{[1 - \pi + \pi F_l(u_l)]^2} \frac{\phi_l}{u_l - v_l} + \frac{\bar{\Pi}}{1 - \pi + \pi F_l(u_l)} \frac{-\phi_l}{[u_l - v_l]^2},$$

which implies that U_h is concave when $\phi_l < 0$. However, the existence of the pooling region implies that $U_h'^{+}(u_2) \geqslant 1 \geqslant U_h'^{-}(u_1)$, which contradicts the concavity of U_h given that $u_1 < u_2$. Hence, either $u_1 = \min \operatorname{Supp}(F_l)$ or $u_2 = \max \operatorname{Supp}(F_l)$. ■

Lemma 13. *If $\phi_l < 0$ and an equilibrium features $[u_1, u_2] \subseteq \operatorname{Supp}(F_l)$ such that $U_h(u_l) > u_l$ for $u_l \in (u_1, u_2)$, then $u_2 = \max \operatorname{Supp}(F_l)$.*

Proof. Suppose by way of contradiction that an equilibrium features separation ($U_h(u_l) > u_l$) on an interval $[u_1, u_2] \subseteq \operatorname{Supp}(F_l)$ with $u_2 < \max \operatorname{Supp}(F_l)$. Then there must exist a pooling interval $[u_2, \bar{u}]$ for some $\bar{u}$. Since $u_2 > \min \operatorname{Supp}(F_l)$, Lemma 12 implies that $\bar{u} = \max \operatorname{Supp}(F_l)$. Since the conjectured equilibrium features separation in $[u_1, u_2]$ with $U_h(u_l) \to u_l$ as $u_l \to u_2$, we must have $U_h'^{-}(u_2) \leqslant 1$. Since the conjectured equilibrium satisfies

$$\frac{\pi f_l(u_l)}{1 - \pi + \pi F_l(u_l)} = \frac{-\phi_l}{u_l - v_l}$$

on the interval $[u_1, u_2]$, $U_h'(u_2) \leqslant 1$ implies

$$\frac{1}{1 - \mu_l \phi_l} \left[-\mu_l \phi_l + \frac{\bar{\Pi}}{1 - \pi + \pi F_l(u_2)} \frac{-\phi_l}{u_2 - v_l} \right] \leqslant 1$$

or

$$-\phi_l \bar{\Pi} \leqslant [1 - \pi + \pi F_l(u_2)] (u_2 - v_l).$$

Since $u_2 < \bar{u}$, $F(u_2) < 1$ so that

$$-\phi_l \bar{\Pi} < u_2 - v_l. \tag{27}$$

Moreover, Lemma 11 ensures that the best equilibrium menu is pooling with utility $\bar{u}$ and, therefore, equilibrium profits satisfy $\bar{\Pi} = \bar{v} - \bar{u}$. Using the fact that $u_2 < \bar{u}$, substituting for $\bar{\Pi}$ in (27), and rearranging terms, we obtain

$$0 < \phi_l - \frac{v_l - \bar{u}}{\bar{v} - \bar{u}}. \tag{28}$$

We will show that (28) implies that a cream-skimming deviation must be a profitable deviation from the best (pooling) menu, yielding the necessary contradiction. Since the conjectured equilibrium features pooling in the interval $[u_2, \bar{u}]$, for u_l in this interval, the equilibrium satisfies

$$(1 - \pi + \pi F_l(u_l))(\bar{v} - u_l) = (1 - \pi)(\bar{v} - \bar{u})$$

so that

$$f_l(u_l) = \frac{1-\pi+\pi F_l(u_l)}{\pi(\bar{v}-u_l)}. \tag{29}$$

Consider then a cream-skimming deviation of the form $(u_l', u_h') = (\bar{u}-\varepsilon, \bar{u})$, which yields profits equal to

$$(1-\pi+\pi F_l(\bar{u}-\varepsilon))\mu_l(v_l-\bar{u}+\varepsilon) + (1-\pi+\pi F_h(\bar{u}))\mu_h \Pi_h(\bar{u}-\varepsilon, \bar{u}). \tag{30}$$

Differentiating (30) with respect to ε and evaluating it at $\varepsilon = 0$, we obtain

$$(1-\pi+\pi F_l(\bar{u}))\mu_l - \pi f_l(\bar{u})\mu_l(v_l-\bar{u}) - (1-\pi+\pi F_h(\bar{u}))\mu_h \frac{v_h-c_h}{c_h-c_l},$$

which, given that $F_l(\bar{u}) = 1$ and $f_l(\bar{u}) = 1/[\pi(\bar{v}-\bar{u})]$, can be written as

$$\mu_l\left[\phi_l - \frac{v_l-\bar{u}}{\bar{v}-\bar{u}}\right] > 0, \tag{31}$$

where the inequality follows from (28). Hence, this cream-skimming deviation strictly increases the buyers' profits relative to the conjectured equilibrium level, a contradiction. ■

Since the only possible equilibria when $\phi_l < 0$, then, are fully separating (except at the upper bound of the support of F_l), fully pooling, or mixed, we need only prove that only one of these equilibria may exist for any value of ϕ_l. We have already shown in the proof of Proposition 4 that $\phi_2 < \phi_1 < 0$. Recall that a necessary condition for a fully pooling equilibrium is that $\phi_l \leqslant \phi_2$. Hence, there is no fully pooling equilibrium when $\phi_l > \phi_2$. Similarly, a necessary condition for a fully separating equilibrium is that $\phi_l \geqslant \phi_1$ so that when $\phi_l < \phi_1$, no fully separating equilibrium exists. This means that in the interval $\phi_2 < \phi_l < \phi_1$, the only possible equilibrium is a mixed equilibrium. Moreover, the threshold in the mixed equilibrium is interior to the support of F_l only if ϕ_l lies between ϕ_2 and ϕ_1. Hence, at most one of these types of equilibria may exist for any value of $\phi_l < 0$, proving that the equilibrium described in Proposition 13 is unique. ■

A.3 Proofs from Section 5

A.3.1 Proof of Proposition 5

In a slight abuse of notation, we write welfare as a function of p,

$$W(p, \mu_h) = (1-\mu_h)(v_l-c_l) + \mu_h\left[(1-p)X_1(p) + pX_2(p)\right], \tag{32}$$

where

$$X_n(p) = (v_h-c_h)\int_{c_l}^{\bar{u}(\pi(p))} x_h(u_l)(v_h-c_h)\,d\left(F_l^n(u_l, \pi(p)\right)$$

and

$$\pi(p) = \frac{2p}{1+p}. \tag{33}$$

Several facts follow immediately from our characterization of equilibrium. First, note that $X_1(0) = X_2(0) = 1$ when $\phi_l < 0$, which implies immediately that welfare is (weakly) maximized at $p = \pi(0) = 0$ in this region of the parameter space. Second, note that $X_1(0) = X_2(0) = 0$ when $\phi_l > 0$, while $X_n(p) > 0$ for all $p \in (0, 1]$. Hence, welfare is minimized at $p = \pi(0) = 0$ in this region of the parameter space. To show that welfare is maximized at an interior value of π when $\phi_l > 0$, we will prove that $\lim_{p\to 1} W_p(p, \mu_h) < 0$.

To this end, first note that

$$\frac{1}{\mu_h} W_p(p, \mu_h) = X_2(p) - X_1(p) + pX_2'(p) + (1-p)X_1'(p).$$

In what follows, we will prove a sequence of results:

1. $\lim_{p\to 1} X_2(p) - X_1(p) = 0$;
2. $\lim_{p\to 1}(1-p)X_1'(p) = 0$;
3. $\lim_{p\to 1} pX_2'(p) < 0$.

The first result follows immediately from the fact that $F_l(u_l)$ converges to a degenerate distribution at $p = \pi(1) = 1$. To prove the second result, we first integrate $X_1'(p)$ by parts:

$$\begin{aligned}
\frac{1}{v_h - c_h}(1-p)X_1'(p) &= (1-p)\frac{d}{dp}\int_{c_l}^{\bar{u}(\pi(p))} x_h(u_l) d\left(F_l(u_l, \pi(p)\right) \\
&= (1-p)\frac{d}{dp}\left[x_h(\bar{u}_l(\pi(p))) - \int_{c_l}^{\bar{u}_l(\pi(p))} x_h'(u_l) F_l(u_l; \pi(p)) du_l\right] \\
&= -(1-p)\int_{c_l}^{\bar{u}_l(\pi(p))} x_h'(u_l)\frac{dF_l(u_l;\pi(p))}{d\pi}\frac{d\pi}{dp} du_l.
\end{aligned}$$

From the definition of $F(u_l)$ in (1), we have

$$\frac{dF_l(u_l;\pi(p))}{d\pi} = -\frac{F_l(u_l;\pi)}{\pi(1-\pi)}.$$

Therefore,

$$\frac{1}{v_h - c_h}(1-p)X_1'(p) = \frac{(1-p)}{\pi(1-\pi)}\frac{d\pi(p)}{dp}\int_{c_l}^{\bar{u}(\pi(p))} x_h'(u_l)F_l(u_l;\pi)du_l.$$

Using (33), we obtain

$$\begin{aligned}
\frac{1}{v_h - c_h}(1-p)X_1'(p) &= \frac{2}{\pi(2-\pi)}\frac{2}{(1+p)^2}\int_{c_l}^{\bar{u}(\pi(p))} x_h'(u_l)F_l(u_l;\pi)du_l \\
&= \frac{2}{\pi(2-\pi)}\frac{2}{(1+p)^2}\left[x_h(\bar{u}(\pi(p))) - \int_{c_l}^{\bar{u}(\pi(p))} x_h(u_l)dF_l(u_l;\pi)\right].
\end{aligned}$$

Since F_l becomes degenerate as $\pi \to 1$ and $\lim_{p\to 1}\pi = 1$, this final results implies

$$\lim_{p\to 1}(1-p)X_1'(p) = (v_h - c_h)\times 2 \times \frac{1}{2}\times 0 = 0.$$

This completes the proof of the second claim above.

To prove the third result, we first integrate $X_2'(p)$ by parts and differentiate:

$$\begin{aligned}
\frac{1}{v_h - c_h} p X_2'(p) &= p \frac{d}{dp} \int_{c_l}^{\bar{u}(\pi(p))} x_h(u_l) d\left(F_l^2(u_l, \pi(p))\right) \\
&= p \frac{d}{dp} \left[x_h(\bar{u}_l(\pi(p))) - \int_{c_l}^{\bar{u}_l(\pi(p))} x_h'(u_l) \left(F_l^2(u_l; \pi(p))\right) du_l \right] \\
&= -p \int_{c_l}^{\bar{u}_l(\pi(p))} x_h'(u_l) \frac{dF_l^2(u_l; \pi(p))}{d\pi} \frac{d\pi}{dp} du_l.
\end{aligned}$$

Since

$$\frac{d}{d\pi} F_l^2(u_l; \pi) = 2F_l(u_l; \pi) \frac{dF_l(u_l; \pi(p))}{d\pi} = -\frac{2}{\pi(1-\pi)} F_l^2(u_l; \pi), \tag{34}$$

we have

$$\begin{aligned}
\frac{1}{v_h - c_h} p X_2'(p) &= \frac{2p}{\pi(1-\pi)} \frac{d\pi(p)}{dp} \int_{c_l}^{\bar{u}_l(\pi(p))} x_h'(u_l) F_l^2(u_l; \pi(p)) du_l \\
&= \frac{2}{(2-\pi)(1-\pi)} \frac{2}{(1+p)^2} \int_{c_l}^{\bar{u}_l(\pi(p))} x_h'(u_l) F_l^2(u_l; \pi(p)) du_l \\
&= \frac{2}{(2-\pi)(1-\pi)} \frac{2}{(1+p)^2} \left[x_h(\bar{u}_l(\pi(p))) - \int_{c_l}^{\bar{u}_l(\pi(p))} x_h(u_l) d\left(F_l^2(u_l; \pi(p))\right) \right].
\end{aligned}$$

To prove the result, we will show that

$$\lim_{\pi \to 1} \frac{1}{1-\pi} \left[x_h(\bar{u}_l(\pi)) - \int_{c_l}^{\bar{u}_l(\pi)} x_h(u_l) d\left(F_l^2(u_l; \pi)\right) \right] < 0.$$

Define $H(\pi)$ as

$$H(\pi) = x_h(\bar{u}_l(\pi)) - \int_{c_l}^{\bar{u}_l(\pi)} x_h(u_l) d\left(F_l^2(u_l; \pi)\right).$$

Since $\lim_{\pi \to 1} H(\pi) = \lim_{\pi \to 1} 1 - \pi = 0$, we will apply L'Hopital's rule:

$$\lim_{\pi \to 1} \frac{H(\pi)}{1-\pi} = -\lim_{\pi \to 1} H'(\pi).$$

Next, using integration by parts, we have

$$H(\pi) = \int_{c_l}^{\bar{u}_l(\pi)} x_h'(u_l) F_l^2(u_l; \pi) du_l.$$

Therefore, using (34), we have

$$\begin{aligned}
H'(\pi) &= x_h'(\bar{u}_l(\pi)) \frac{d\bar{u}_l}{d\pi} + \int_{c_l}^{\bar{u}_l(\pi)} x_h'(u_l) \frac{dF_l^2(u_l; \pi)}{d\pi} du_l \\
&= x_h'(\bar{u}_l(\pi)) \frac{d\bar{u}_l}{d\pi} - \frac{2}{\pi(1-\pi)} \int_{c_l}^{\bar{u}_l(\pi)} x_h'(u_l) F_l^2(u_l; \pi) du_l \\
&= x_h'(\bar{u}_l(\pi)) \frac{d\bar{u}_l}{d\pi} - \frac{2}{\pi} \frac{H(\pi)}{(1-\pi)}.
\end{aligned}$$

Therefore,

$$\lim_{\pi\to 1}\frac{H(\pi)}{1-\pi} = -\lim_{\pi\to 1} x_h'(\bar{u}_l(\pi))\frac{d\bar{u}_l}{d\pi} + 2\lim_{\pi\to 1}\frac{H(\pi)}{1-\pi},$$

so that, rearranging the terms, we have

$$\lim_{\pi\to 1}\frac{H(\pi)}{1-\pi} = \lim_{\pi\to 1} x_h'(\bar{u}_l(\pi))\frac{d\bar{u}_l}{d\pi}.$$

We now prove that $\lim_{\pi\to 1} x_h'(\bar{u}_l(\pi))\frac{d\bar{u}_l}{d\pi} < 0$. Using the fact that

$$x_h(u_l) = \frac{1}{\mu_h(v_h - c_l)}\left\{(1-\mu_h)(v_l - c_l)^{1-\phi}(v_l - u_l)^{\phi} - (1-\mu_h)v_l + u_l - \mu_h c_l\right\},$$

we have

$$x_h'(u_l) = \frac{1}{\mu_h(v_h - c_l)}\left(1 - \phi(1-\mu_h)(v_l - c_l)^{1-\phi}(v_l - u_l)^{\phi-1}\right).$$

Next, since $\bar{u}_l(\pi)$ satisfies

$$1 = (1-\pi)\left(\frac{v_l - c_l}{v_l - \bar{u}_l(\pi)}\right)^{\phi},$$

we have

$$v_l - \bar{u}_l(\pi) = (1-\pi)^{\frac{1}{\phi}}(v_l - c_l),$$

which implies

$$\frac{d\bar{u}_l(\pi)}{d\pi} = \frac{1}{\phi}(1-\pi)^{\frac{1}{\phi}-1}(v_l - c_l)$$

and

$$x_h'(\bar{u}_l(\pi)) = \frac{1}{\mu_h(v_h - c_l)}\left(1 - \phi(1-\mu_h)(1-\pi)^{\frac{\phi-1}{\phi}}\right).$$

Combining these results, we find

$$x_h'(\bar{u}_l(\pi))\frac{d\bar{u}_l}{d\pi} = \frac{v_l - c_l}{\mu_h(v_h - c_l)}\left[\frac{(1-\pi)^{\frac{1}{\phi}-1}}{\phi} - (1-\mu_h)\right]$$

and hence

$$\lim_{\pi\to 1} x_h'(\bar{u}_l(\pi))\frac{d\bar{u}_l}{d\pi} = -\frac{(1-\mu_h)(v_l - c_l)}{\mu_h(v_h - c_l)} < 0.$$

■

A.3.2 Proof of Proposition 6

Proof. We start with the form of $W(p, \mu_h)$ given by (32) and express this instead as a function of π. Tedious, but straightforward calculations can be used to show

$$\begin{aligned} W(\pi, \mu_h) &= (1-\mu_h)(v_l - c_l)\left[1 + \frac{(v_h - c_h)}{(v_h - c_l)}\frac{2(1-\pi)}{(2-\pi)}\right] + \mu_h\left[c_h + \frac{(v_h - c_h)}{(v_h - c_l)}(v_l - c_l)\right] \\ &\quad + \frac{(v_h - c_h)}{(v_h - c_l)}\frac{2(1-\pi)^2}{\pi(2-\pi)}(v_l - c_l)\frac{\phi(\mu_h)}{1-2\phi(\mu_h)}\left((1-\pi)^{\frac{1-2\phi(\mu_h)}{\phi(\mu_h)}} - 1\right). \end{aligned}$$

Then $W_{\mu_h}(\pi, \mu_h)$ satisfies

$$W_{\mu_h}(\pi, \mu_h) = \Theta + \frac{(\nu_h - c_h)(\nu_l - c_l)}{\nu_h - c_l} \frac{2(1-\pi)^2}{\pi(2-\pi)} \frac{d}{d\mu_h} \frac{\phi(\mu_h)}{1 - 2\phi(\mu_h)} \left[(1-\pi)^{\frac{1}{\phi(\mu_h)}-2} - 1\right], \tag{35}$$

where

$$\Theta = c_h - (\nu_l - c_l) + \frac{\nu_h - c_h}{\nu_h - c_l}(\nu_l - c_l)\frac{\pi}{2-\pi}.$$

We will argue that when π is sufficiently small, then

$$\lim_{\mu_h \to 0} W_{\mu_h}(\pi, \mu_h) < \lim_{\mu_h \to \mu_0} W_{\mu_h}(\pi, \mu_h), \tag{36}$$

where μ_0 is the value of μ_h such that $\phi(\mu_0) = 0$. Inequality (36) implies that the W_{μ_h} must be increasing on an interval of μ_h[2]; that is, W must be convex on an interval of μ_h. In contrast, the inequality above is reversed when π is sufficiently close to 1.

Let

$$M(\pi, \mu_h) = \frac{d}{d\mu_h} \frac{\phi(\mu_h)}{1 - 2\phi(\mu_h)} \left[(1-\pi)^{\frac{1}{\phi(\mu_h)}-1} - 1\right]$$

and

$$G(\pi) = \lim_{\mu_h \to 0} M(\pi, \mu_h) - \lim_{\mu_h \to \mu_0} M(\pi, \mu_h).$$

Since the term multiplying $M(\pi, \mu_h)$ in (35) is positive, it suffices to show that $G(\pi) < 0$ for π close to 0 and $G(\pi) > 0$ for π close to 1. Note that

$$\lim_{\mu_h \to 0} M(\pi, \mu_h) = -\left(\frac{\nu_h - c_h}{c_h - c_l}\right) \frac{\pi + \log(1-\pi)}{1-\pi}$$

and

$$\lim_{\mu_h \to \mu_0} M(\pi, \mu_h) = \phi'(\mu_0)\left[-1 - \log(1-\pi) \lim_{\phi \to 0} \frac{(1-\pi)^{\frac{1}{\phi}-2}}{\phi}\right] = \left(\frac{\nu_h - c_h}{c_h - c_l}\right) \frac{1}{(1-\mu_0)^2}.$$

As a result,

$$G(\pi) = -\left(\frac{\nu_h - c_h}{c_h - c_l}\right) \frac{\pi + \log(1-\pi)}{1-\pi} - \left(\frac{\nu_h - c_h}{c_h - c_l}\right) \frac{1}{(1-\mu_0)^2}$$

It follows that

$$\lim_{\pi \to 0} G(\pi) = -\left(\frac{\nu_h - c_h}{c_h - c_l}\right) \frac{1}{(1-\mu_0)^2}$$

and

$$\lim_{\pi \to 1} G(\pi) = +\infty,$$

which completes the proof. ∎

A.4 Proofs from Section 6

A.4.1 Proof of Proposition 7

Given $\hat{\pi}^1 = \hat{\pi}^2 \equiv \hat{\pi}$, we can use the analysis of our benchmark model to characterize the unique equilibrium. In particular, substituting $\hat{\pi} = \pi$, Propositions 2 and 3 characterize the equilibrium offer distributions, $\{F_j(u_j)\}$, along with equilibrium profits, which we denote by $\Pi^\star(\pi)$. For any $\phi_l < 1$, equilibrium

[2] It is straightforward to show that $W_{\mu_h}(\pi, \mu_h)$ is continuous.

profits are continuous and strictly decreasing in π, with $\Pi^\star(0) > 0$ and $\Pi^\star(1) = 0$. By assumption, $C'(\pi)$ is a continuous, strictly increasing function with $C'(0) = 0$ and $C'(1) > 0$, so that there is a unique solution to the first-order condition $C'(\pi) = \Pi^\star(\pi)$. ■

A.4.2 Proof of Lemma 5

Consider first the case of $\phi_l \geqslant 0$. In this region of the parameter space, $\pi^\star$ satisfies

$$C'(\pi^\star) - (1-\mu_h)(1-\pi^\star)(v_l - c_l) = 0, \tag{37}$$

and hence clearly $\frac{d\pi^\star}{d\mu_h} < 0$.

Next consider the case of $\phi_l < 0$. If $\phi_l \leqslant \phi_1$, where $\phi_1 < 0$ is defined in (8), then $\pi^\star$ satisfies

$$C'(\pi^\star) - (1-\pi^\star)(\mu_h v_h + \mu_l v_l - c_h) = 0, \tag{38}$$

and hence clearly $\frac{d\pi^\star}{d\mu_h} > 0$. The more difficult case is when $\phi_1 < \phi_l < 0$. In this case, $\pi^\star$ satisfies

$$C'(\pi^\star) - (1-\pi^\star)\left[\mu_l(v_l - \hat{u}_l) + \mu_h \Pi_h(\hat{u}_l, c_h)\right] = 0, \tag{39}$$

where $\hat{u}_l$ satisfies $\Upsilon(\hat{u}_l, \mu_h) = 0$, with

$$\Upsilon(\hat{u}_l, \mu_h) = \bar{v} - \left[v_l + (1-\pi) g(\mu_h, \pi)(\hat{u}_l - v_l)\right] - (1-\pi)\left[\mu_l(v_l - \hat{u}_l) + \mu_h \Pi_h(\hat{u}_l, c_h)\right] \tag{40}$$

and

$$g(\mu_h, \pi) = (1-\pi)^{\frac{1}{\phi_l} - 1}.$$

To prove that $\frac{d\pi^\star}{d\mu_h} > 0$, we will show that

$$\Pi^\star(\pi, \mu_h) = (1-\pi)\left[\mu_l(v_l - \hat{u}_l) + \mu_h \Pi_h(\hat{u}_l, c_h)\right]$$

is increasing in μ_h and decreasing in π. To prove the first result, note that

$$\frac{\partial \Upsilon}{\partial \hat{u}_l} = -\frac{(1-\pi)^{\frac{1}{\phi_l}}}{\mu_l} + (1-\pi)\phi_l < 0,$$

since we are looking at the region with $\phi_l < 0$, and

$$\begin{aligned}
\frac{\partial \Upsilon}{\partial \mu_h} &= (v_h - v_l) - (1-\pi)\left[(\hat{u}_l - v_l)\left(\frac{\partial g}{\partial \mu_h} + 1\right) + \Pi_h(\hat{u}_l, c_h)\right] \\
&\geqslant (v_h - v_l) - (1-\pi)\left[(\hat{u}_l - v_l) + \Pi_h(\hat{u}_l, c_h)\right] \\
&= \pi(v_h - v_l) + (1-\pi)(c_h - \hat{u}_l)\left(\frac{v_h - c_l}{c_h - c_l}\right) \quad \geqslant 0,
\end{aligned}$$

where we use that $\frac{\partial g}{\partial \mu_h} < 0$ in the first inequality and $c_h \geqslant \hat{u}_l$ in the last. Hence, $\frac{d\hat{u}_l}{d\mu_h} \geqslant 0$ and

$$\frac{\partial \bar{\Pi}^\star}{\partial \mu_h} = (1-\pi)\left[\Pi_h(\hat{u}_l, c_h) + (\hat{u}_l - v_l) - \mu_l \frac{d\hat{u}_l}{d\mu_h}\phi_l\right] \geqslant 0.$$

To show that $\Pi^\star$ is decreasing in π, we must show that

$$\frac{1}{\phi_l}(1-\pi)^{\frac{1}{\phi_l}-1}(\underline{u} - v_l) - (1-\pi)^{\frac{1}{\phi_l}}\frac{d\hat{u}_l}{d\pi} \leqslant 0,$$

or

$$\frac{d\hat{u}_l}{d\pi} \geqslant \frac{\hat{u}_l - \nu_l}{\phi_l (1-\pi)}. \tag{41}$$

Solving (40) explicitly for $\hat{u}_l$ yields

$$\begin{aligned}\hat{u}_l &= \frac{\bar{\nu} - \nu_l + (1-\pi)^{\frac{1}{\phi_l}} \nu_l - (1-\pi)\left(\bar{\nu} - \mu_h \frac{\nu_h - c_l}{c_h - c_l} c_h\right)}{(1-\pi)^{\frac{1}{\phi_l}} - \mu_l \phi_l (1-\pi)} \\ &= \nu_l + \frac{\mu_h (\nu_h - \nu_l) - (1-\pi)\, \mu_h \frac{(\nu_l - c_l)(\nu_h - c_h)}{c_h - c_l}}{(1-\pi)^{\frac{1}{\phi_l}} - \mu_l \phi_l (1-\pi)}\end{aligned}$$

so that

$$\frac{d\hat{u}_l}{d\pi} = \frac{\mu_h \frac{(\nu_l - c_l)(\nu_h - c_h)}{c_h - c_l}}{(1-\pi)^{\frac{1}{\phi_l}} - \mu_l \phi_l (1-\pi)} - (\hat{u}_l - \nu_l) \frac{-\frac{1}{\phi_l}(1-\pi)^{\frac{1}{\phi_l}-1} + \mu_l \phi_l}{(1-\pi)^{\frac{1}{\phi_l}} - \mu_l \phi_l (1-\pi)}.$$

Thus, we have to show that

$$\begin{aligned}& \frac{\mu_h \frac{(\nu_l - c_l)(\nu_h - c_h)}{c_h - c_l}}{(1-\pi)^{\frac{1}{\phi_l}} - \mu_l \phi_l (1-\pi)} - (\hat{u}_l - \nu_l) \frac{-\frac{1}{\phi_l}(1-\pi)^{\frac{1}{\phi_l}-1} + \mu_l \phi_l}{(1-\pi)^{\frac{1}{\phi_l}} - \mu_l \phi_l (1-\pi)} \geqslant \frac{\hat{u}_l - \nu_l}{\phi_l (1-\pi)} \\ \Leftrightarrow\ & \frac{\mu_h \frac{(\nu_l - c_l)(\nu_h - c_h)}{c_h - c_l}}{(1-\pi)^{\frac{1}{\phi_l}} - \mu_l \phi_l (1-\pi)} \geqslant \frac{(\hat{u}_l - \nu_l)\, \mu_l (\phi_l - 1)}{\left((1-\pi)^{\frac{1}{\phi_l}} - \mu_l \phi_l (1-\pi)\right)}.\end{aligned}$$

Again, since $\hat{u}_l \geqslant \nu_l$ and $\phi_l \leqslant 0$, the right-hand side of the inequality above is negative and the left-hand side is positive. This completes the proof.

A.4.3 Proof of Lemma 6

Let

$$X(\pi) = \left[2\pi(1-\pi) + \pi^2\right] (\mu_h X_h(\pi)(\nu_h - c_h) + \mu_l(\nu_l - c_l)) \tag{42}$$

denote the gains from trade realized in a symmetric equilibrium with $\pi^\star = \pi$. Welfare is defined as

$$W(\tau) = -2Ac\,(\pi(\tau)) + X\,(\pi(\tau))\,, \tag{43}$$

so that

$$W'(\tau) = \left[X'(\pi) - 2Ac'(\pi)\right] \frac{d\pi}{d\tau}. \tag{44}$$

Since

$$\lim_{A \to 0} \pi^\star = 1,$$

and

$$\lim_{\pi \to 1} X(\pi) < 0,$$

there exists $\widetilde{A} > 0$ such that $X'(\pi) < 0$ whenever $A < \widetilde{A}$. Since $\frac{d\pi}{d\tau} < 0$, it follows immediately from (44) that $W'(\tau) > 0$ in this region. ■

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A.5 Proofs from Section 7

A.5.1 Poisson Meeting Technology

Since $nP_n = \lambda Q_n$ for all $n \in \mathbb{N}$, we have:

$$Q_n(\alpha) = \frac{e^{-\alpha}\alpha^{n-1}}{(n-1)!}$$

and $Q_0(\alpha) = 1 - \sum_{n=1}^{\infty} Q_n(\alpha) = 0$. From the definition of $\tilde{\pi}$, we have $\tilde{\pi} = 1 - Q_1(\alpha)$ and substituting into (26) implies

$$\begin{aligned} G_l(u_l;\alpha) &= \frac{1}{1-Q_1(\alpha)} \sum_{n=2}^{\infty} Q_n(\alpha) F_l^{n-1}(u_l;\alpha) \\ &= \frac{Q_1(\alpha)}{1-Q_1(\alpha)} \left[e^{\alpha F_l(u_l;\alpha)} - 1 \right]. \end{aligned} \tag{45}$$

Next, using the solution to the differential equation (27),

$$1 - \tilde{\pi} + \tilde{\pi} G_l(u_l;\alpha) = (1-\tilde{\pi}) \left(\frac{v_l - u_l}{v_l - c_l} \right)^{-\phi_l},$$

one can show that

$$G_l(u_l;\alpha) = \frac{Q_1(\alpha)}{1-Q_1(\alpha)} \left[\left(\frac{v_l - u_l}{v_l - c_l} \right)^{-\phi_l} - 1 \right]. \tag{46}$$

Combining (45) and (46), we obtain

$$F_l(u_l;\alpha) = \frac{-\phi_l}{\alpha} \log \left(\frac{v_l - u_l}{v_l - c_l} \right).$$

Note also that $F_l(\bar{u}_l;\alpha) = 1$ implies

$$\frac{v_l - \bar{u}_l}{v_l - c_l} = e^{-\frac{\alpha}{\phi_l}}, \tag{47}$$

and

$$f_l(u_l;\alpha) = \frac{\phi_l}{\alpha} (v_l - u_l)^{-1},$$

which we use below.

Next, we evaluate the utilitarian welfare measure given the Poisson meeting technology:

$$\begin{aligned} &\sum_{n=1}^{\infty} P_n(\alpha) \left[\mu_h(v_h - c_h) \int x_h(u_l) d(F_l^n(u_l;\alpha)) + \mu_l(v_l - c_l) \right] \\ &= \mu_h(v_h - c_h) \int x_h(u_l) \sum_{n=1}^{\infty} nP_n(\alpha) F_l^{n-1}(u_l;\alpha) f_l(u_l;\alpha) du_l + \mu_l(v_l - c_l) \sum_{n=1}^{\infty} P_n(\alpha) \\ &= \mu_h(v_h - c_h) \int x_h(u_l) \sum_{n=1}^{\infty} nP_n(\alpha) F_l^{n-1}(u_l;\alpha) f_l(u_l;\alpha) du_l + \mu_l(v_l - c_l)(1 - e^{-\alpha}). \end{aligned}$$

Consider

$$\hat{W}(\alpha) = \int x_h(u_l) \sum_{n=1}^{\infty} nP_n(\alpha) F_l^{n-1}(u_l;\alpha) f_l(u_l;\alpha) du_l. \tag{48}$$

Substituting for $nP_n(\alpha)$, and using (26) and (46), we obtain

$$\begin{aligned}\hat{W}(\alpha) &= \int x_h(u_l)\alpha \sum_{n=1}^{\infty} Q_n(\alpha)F_l^{n-1}(u_l;\alpha)f_l(u_l;\alpha)du_l \\ &= \int x_h(u_l)\alpha\left[Q_1(\alpha)+(1-Q_1(\alpha))G_l(u_l;\alpha)\right]f_l(u_l;\alpha)du_l \\ &= \int x_h(u_l)\alpha Q_1(\alpha)\left(\frac{v_l-u_l}{v_l-c_l}\right)^{-\phi_l} f_l(u_l;\alpha)du_l.\end{aligned}$$

Substituting for $f_l(u_l)$ and $x_h(u_l)$ and rearranging terms yields

$$\hat{W}(\alpha) = Q_1(\alpha)\frac{\phi_l(v_l-c_l)^{\phi_l}}{\mu_h(v_h-c_l)}\int (v_l-u_l)^{-1-\phi_l}\left[\mu_l(v_l-c_l)^{1-\phi_l}(v_l-u_l)^{\phi_l}-(v_l-u_l)+\mu_h(v_l-c_l)\right]du_l,$$

where the limits of integration are c_l and $\bar{u}_l(\alpha)$. Applying tedious but straightforward calculus to compute the integral yields

$$\hat{W}(\alpha) = e^{-\alpha}\frac{\phi_l}{\mu_h(v_h-c_l)}\left[\mu_l(v_l-c_l)\frac{\alpha}{\phi_l}+\frac{v_l-c_l}{1-\phi_l}\left(e^{-\alpha\frac{1-\phi_l}{\phi_l}}-1\right)\right]+\frac{v_l-c_l}{v_h-c_l}-e^{-\alpha}\frac{v_l-c_l}{v_h-c_l}.$$

Evaluating welfare as a function of α then implies

$$\begin{aligned}W(\alpha) =& \mu_h(v_h-c_h)\left[\frac{v_l-c_l}{v_h-c_l}+\frac{\mu_l(v_l-c_l)}{\mu_h(v_h-c_l)}\alpha e^{-\alpha}+\frac{\phi_l}{1-\phi_l}\frac{v_l-c_l}{\mu_h(v_h-c_l)}\left(e^{-\frac{\alpha}{\phi_l}}-e^{-\alpha}\right)-e^{-\alpha}\frac{v_l-c_l}{v_h-c_l}\right] \\ &+(1-e^{-\alpha})\mu_l(v_l-c_l).\end{aligned}$$

Differentiating welfare with respect to α and rearranging terms, we obtain

$$W'(\alpha) = e^{-\alpha}\frac{(v_h-c_h)(v_l-c_l)}{v_h-c_l}\left[\mu_l(1-\alpha)-\frac{1}{(1-\phi_l)}e^{-\alpha\frac{1-\phi_l}{\phi_l}}+\frac{\phi_l}{1-\phi_l}+\mu_h+\mu_l\frac{v_h-c_h}{v_h-c_l}\right].$$

Let $H(\alpha)$ denote the term in brackets in the equation above. Since $H(\alpha)$ is a strictly concave function with $H(0) > 0$ and $\lim_{\alpha\to\infty} H(\alpha) = -\infty$, there exists a unique α^* such that for all $\alpha > \alpha^*$, $H(\alpha) < 0$. Hence, for all finite $\alpha > \alpha^*$, $W'(\alpha) < 0$.

A.5.2 Geometric Meeting Technology

For the Geometric meeting technology with $\lambda(\alpha) = \alpha/(1-\alpha)$, we have

$$Q_n(\alpha) = (1-\alpha)^2 n\alpha^{n-1}$$

and $Q_0(\alpha) = 0$. Much as in the Poisson case, one can use (26) and (27) to show

$$F_l(u_l;\alpha) = \frac{1}{\alpha}\left[1-\left(\frac{v_l-u_l}{v_l-c_l}\right)^{\frac{\phi_l}{2}}\right],$$

so that

$$f_l(u_l;\alpha) = \frac{\phi_l}{2\alpha}\left(\frac{v_l-u_l}{v_l-c_l}\right)^{\frac{\phi_l}{2}}\frac{1}{v_l-u_l}$$

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and the upper bound $\bar{u}_l(\alpha)$ satisfies

$$\left(\frac{v_l - \bar{u}_l(\alpha)}{v_l - c_l}\right)^{\frac{\phi_l}{2}} = 1 - \alpha.$$

Next, we evaluate the utilitarian welfare measure given the Geometric meeting technology:

$$\begin{aligned} &\sum_{n=1}^{\infty} P_n(\alpha)\left[\mu_h(v_h - c_h)\int x_h(u_l)d(F_l^n(u_l;\alpha)) + \mu_l(v_l - c_l)\right] \\ &= \mu_h(v_h - c_h)\int x_h(u_l)\sum_{n=1}^{\infty} nP_n(\alpha)F_l^{n-1}(u_l;\alpha)f_l(u_l;\alpha)du_l + \mu_l(v_l - c_l)\alpha. \end{aligned}$$

Consider $\hat{W}(\alpha)$, as defined in (48). Using similar steps to those we used above, one can show that

$$\begin{aligned} \hat{W}(\alpha) &= (1-\alpha)\frac{\phi_l(v_l - c_l)^{\frac{\phi_l}{2}}}{2\mu_h(v_h - c_l)}\int_{c_l}^{\bar{u}_l(\alpha)}(v_l - u_l)^{-1-\frac{\phi_l}{2}}\left[\mu_l(v_l - c_l)^{1-\phi_l}(v_l - u_l)^{\phi_l} - (v_l - u_l) + \mu_h(v_l - c_l)\right]du_l \\ &= \frac{\phi_l(1-\alpha)}{2\mu_h(v_h - c_l)}(v_l - c_l)\left\{\mu_l\alpha\frac{2}{\phi_l} + \frac{1}{1-\phi_l/2}\left[(1-\alpha)^{2/\phi_l - 1} - 1\right] + \mu_h\frac{\alpha}{1-\alpha}\frac{2}{\phi_l}\right\}. \end{aligned}$$

Therefore, welfare as a function of α is given by

$$\begin{aligned} W(\alpha) = &\frac{\phi_l(1-\alpha)(v_h - c_h)(v_l - c_l)}{2(v_h - c_l)}\left\{\mu_l\alpha\frac{2}{\phi_l} + \frac{1}{1-\phi_l/2}\left[(1-\alpha)^{2/\phi_l - 1} - 1\right] + \mu_h\frac{\alpha}{1-\alpha}\frac{2}{\phi_l}\right\} \\ &+ \alpha\mu_l(v_l - c_l). \end{aligned}$$

Differentiating with respect to α and rearranging terms, we obtain

$$W'(\alpha) = \mu_l(v_l - c_l) + \frac{(v_h - c_h)(v_l - c_l)}{(v_h - c_l)}\left[\mu_l(1-2\alpha) - \frac{1}{1-\phi_l/2}(1-\alpha)^{\frac{2}{\phi_l}-1} + \frac{\phi_l/2}{1-\phi_l/2} + \mu_h\right].$$

Note that $W'(0) = \mu_l(v_l - c_l)$ and

$$W'(1) = \mu_l(v_l - c_l)\frac{c_h - c_l}{v_h - c_l} + \frac{(v_h - c_h)(v_l - c_l)}{(v_h - c_l)}\left[\mu_h + \frac{\phi_l/2}{1-\phi_l/2}\right].$$

Since $W'(0) > 0$ and $W'(1) > 0$ and $W'(\alpha)$ is a strictly concave function of α, there exists no $\alpha \in (0,1)$ such that $W'(\alpha) < 0$.

B Constrained Efficiency

In this section, we examine the efficiency properties of equilibrium outcomes. We define the type of seller $i \in [0,1]$ by $\theta_i \in \Theta$, where $\Theta = \{l,h\} \times \{0,1\} \times \{0,1\}$. The first element of θ_i indicates whether the seller has a high- or low-quality good, the second element equals 1 if the seller is matched with buyer 1 and 0 otherwise, and the third element equals 1 if the seller is matched with buyer 2 and 0 otherwise. We let $c : \Theta \to \{c_l, c_h\}$ denote the valuation a seller of type $\theta \in \Theta$ has for her own good, and $v : \Theta \to \{v_l, v_h\}$ denote the buyer's valuation of a good purchased from a seller with type θ. Let $\bar{\theta} : [0,1] \to \Theta$ denote the mapping from sellers to their respective types, with $\bar{\Theta}$ representing the set of all possible mappings, $\bar{\theta}$.

Each buyer's type consists of the set of sellers with whom the buyer is matched. We represent the type of buyer $k \in \mathcal{B} = \{1,2\}$ as a mapping $m^k(i) : [0,1] \to \{0,1\}$. Let $\boldsymbol{m}^k$ denote the mapping $m^k(i)$ and $\mathcal{M}$ denote the set of all possible functions $\boldsymbol{m}^k$.

We model the realization of $\bar{\theta}$ and $\{m^k\}_{k\in\mathcal{B}}$ as the realization of a random variable that is drawn from a known distribution.[3] This ensures that the beliefs of each buyer and seller about the types of other buyers and sellers conditional on knowledge of their own type give rise to well-defined conditional expectations, as discussed in Uhlig (1996).

An allocation is a given by $\left(t_i^k, x_i^k\right)_{k\in\mathcal{B}, i\in[0,1]}$, where $t_i^k \in \mathbb{R}$ is a transfer of numeraire from buyer k to seller i and $x_i^k \in [0,1]$ is the amount of good transferred from seller i to buyer k. An allocation is *feasible* if for all i and k such that $m^k(i) = 0$, the allocation satisfies $t_i^k = x_i^k = 0$ and for all i, $x_i^1 x_i^2 = 0$. The first constraint ensures that transfers of numeraire and goods only occur between matched buyers and sellers, while the second constraint ensures that trade is exclusive.

We consider the class of *direct mechanisms* given by $(\boldsymbol{t}_i^k, \boldsymbol{x}_i^k)_{k\in\mathcal{B}, i\in[0,1]}$, where $\boldsymbol{t}_i^k : \bar{\Theta} \times \mathcal{M}^2 \to \mathbb{R}$ and $\boldsymbol{x}_i^k : \bar{\Theta} \times \mathcal{M}^2 \to [0,1]$.[4]

Constrained Efficiency with Direct Mechanisms. We begin by defining and characterizing incentive compatible direct mechanisms. A direct mechanism is *incentive compatible* if and only if, for all sellers i,

$$\begin{aligned}&\mathbb{E}\left[\sum_{k\in\mathcal{B}}[t_i^k(\theta_i, \theta_{-i}, \boldsymbol{m}^1, \boldsymbol{m}^2) + (1 - x_i^k(\theta_i, \theta_{-i}, \boldsymbol{m}^1, \boldsymbol{m}^2))c(\theta_i)]\right] \\ &\geqslant \mathbb{E}\left[\sum_{k\in\mathcal{B}}[t_i^k(\hat{\theta}_i, \theta_{-i}, \boldsymbol{m}^1, \boldsymbol{m}^2) + (1 - x_i^k(\hat{\theta}_i, \theta_{-i}, \boldsymbol{m}^1, \boldsymbol{m}^2))c(\theta_i)]\right] \quad \forall \hat{\theta}_i \in \Theta, \end{aligned} \tag{49}$$

and, for each buyer $k \in \mathcal{B}$,

$$\begin{aligned}&\mathbb{E}\left[\int_i [x_i^k(\bar{\theta}, \boldsymbol{m}^k, \boldsymbol{m}^{-k})v(\bar{\theta}_i) - t_i^k(\bar{\theta}, \boldsymbol{m}^k, \boldsymbol{m}^{-k})]di\right] \\ &\geqslant \mathbb{E}\left[\int_i [x_i^k(\bar{\theta}, \hat{\boldsymbol{m}}^k, \boldsymbol{m}^{-k})v(\bar{\theta}_i) - t_i^k(\bar{\theta}, \hat{\boldsymbol{m}}^k, \boldsymbol{m}^{-k})]di\right] \quad \forall \hat{\boldsymbol{m}}^k \in \mathcal{M}, \end{aligned} \tag{50}$$

where the conditional expectations in (49) and (50) are taken with respect to other agents' types.

Lastly, a direct mechanism satisfies *individual rationality* if and only if for all sellers i,

$$\mathbb{E}\left[\sum_{k\in\mathcal{B}}[t_i^k(\theta_i, \theta_{-i}, \boldsymbol{m}^1, \boldsymbol{m}^2) + (1 - x_i^k(\theta_i, \theta_{-i}, \boldsymbol{m}^1, \boldsymbol{m}^2))c(\theta_i)]\right] \geqslant V^s(\theta_i), \tag{51}$$

where $V^s(\theta_i)$ denotes the expected value a seller expects to receive in equilibrium, or,

$$V^s(\theta_i) = \begin{cases} \int \left[t_{\theta_{i,1}}(u_l) + c(\theta_i)(1 - x_{\theta_{i,1}}(u_l))\right] dF_l(u_l) & \text{if } m^1(i)m^2(i) = 0 \\ \int \left[t_{\theta_{i,1}}(u_l) + c(\theta_i)(1 - x_{\theta_{i,1}}(u_l))\right] d(F_l(u_l)^2) & \text{if } m^1(i)m^2(i) = 1 \end{cases},$$

and for each buyer $k \in \mathcal{B}$,

$$\mathbb{E}\left[\int_i [x_i^k(\bar{\theta}, \boldsymbol{m}^k, \boldsymbol{m}^{-k})v(\bar{\theta}_i) - t_i^k(\bar{\theta}, \boldsymbol{m}^k, \boldsymbol{m}^{-k})]di\right] \geqslant V^b, \tag{52}$$

where V^b represents the buyer's expected equilibrium value, or

$$V^b = \frac{1}{2-\pi}\sum_{i=l,h}\mu_i\left\{(1-\pi)\int[v_i x_i(u_l) - t_i(u_l)]dF_l(u_l) + \frac{\pi}{2}\int[v_i x_i(u_l) - t_i(u_l)]d(F_l(u_l)^2)\right\}.$$

[3] A complete description of one way to model this aggregate shock and the resulting expectations is available upon request.

[4] The Revelation Principle applies immediately to this environment so that we may restrict attention to direct mechanisms without loss of generality.

Characterization. We proceed by characterizing the set of mechanisms that maximize the sum of buyers' utilities. First, we simplify the set of incentive constraints. Note that each seller's match type—i.e., whether they are matched with buyer 1, buyer 2, or both—is correlated with the buyers' match types. As a result, it is straightforward to design a direct mechanism in which sellers have no incentives to lie about their match type, and buyers' incentive constraints are slack. This allows us to rewrite mechanisms simply as transfers (of the numeraire and the good) for each of the four types of sellers: those with high- or low-quality goods and those matched with one or two buyers.

Imposing symmetry, we redefine the mechanism as $\{t(i,n), x(i,n)\}$ for $i = \{l, h\}$ and $n = \{1, 2\}$ as the expected transfer and trade by a seller with quality i and n offers. Interim incentive compatibility requires, for each (i, n)

$$t(i,n) + (1 - x(i,n))c_i \geqslant t(\hat{i},n) + (1 - x(\hat{i},n))c_i. \tag{53}$$

Individual rationality of the sellers requires

$$t(i,1) + (1 - x(i,1))c_i \geqslant V^s(i,1) = \int [t_i(u_l) + (1 - x_i(u_l))c_i]\, dF_l(u_l), \tag{54}$$

$$t(i,2) + (1 - x(i,2))c_i \geqslant V^s(i,2) = \int [t_i(u_l) + (1 - x_i(u_l))c_i]\, d(F_l(u_l)^2). \tag{55}$$

Buyers' utility associated with any such mechanism satisfies

$$\frac{2}{2-\pi} \sum_{i=l,h} \mu_i \left[(1-\pi)(\nu_i x(i,1) - t(i,1)) + \frac{\pi}{2}(\nu_i x(i,2) - t(i,2))\right]. \tag{56}$$

Thus, a constrained efficient allocation is a feasible allocation which maximizes (56) subject to (53)–(55). It is immediate that such an allocation satisfies $x(l,n) = 1$ for $n = 1, 2$ and

$$t(l,n) = t(h,n) + (1 - x(h,n))c_l.$$

That is, constrained efficient allocations do not distort trade for low-quality sellers (matched with either one or two buyers), and the incentive constraint for low-quality sellers must bind. Moreover, the individual rationality constraints for high-quality sellers necessarily bind. If these constraints did not bind, one could decrease the surplus allocated to sellers of high-quality goods by increasing $x(h,n)$ by ϵ and $t(h,n)$ by ϵc_l. Such a perturbation raises aggregate buyers' payoffs by $\epsilon(\nu_h - c_l)$, preserves incentives, and for ϵ small does not violate individual rationality of high-quality sellers.

We now state our main proposition concerning the efficiency of the equilibrium in our environment.

Proposition 1. *If $\phi_l > 0$ or $\phi_l < \phi_2$, where ϕ_2 is defined in Proposition 4, then the equilibrium is constrained efficient. If $\phi_l \in [\phi_2, 0]$, then the equilibrium is constrained inefficient.*

Proof of Proposition 1. We first prove that equilibrium allocation is constrained efficient when $\phi_l > 0$. To start, note that the individual rationality constraint for low-quality sellers must bind, otherwise one can improve buyers' payoffs by reducing transfers to low-quality sellers and adjusting trade with high-quality sellers to preserve incentive compatibility. Since $\phi_l > 0$, such a perturbation raises buyers' utility. Summarizing the results above, when $\phi_l > 0$ the solution to the program described above must satisfy $x(l,n) = 1$,

$$t(l,n) = V^s(l,n), \tag{57}$$

$$t(h,n) + (1 - x(h,n))c_h = V^s(h,n), \quad \text{and} \tag{58}$$

$$t(l,n) = t(h,n) + (1 - x(h,n))c_l. \tag{59}$$

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We now show that expected volume of trade in a constrained efficient allocation coincides with expected trade in our equilibrium. It is clear that trade by low-quality sellers is the same (since $x(l,n)=1$ for $n=1,2$). To see that trade by high-quality sellers also coincides, first note that (57)–(59) imply

$$V^s(h,n)-V^s(l,n)=[1-x(h,n)]\,(c_h-c_l). \tag{60}$$

Using the definition of $V(i,n)$ in (54)–(55), along with the fact that each menu in equilibrium satisfies the low-quality seller's incentive constraint with equality, we have

$$V^s(h,n)-V^s(l,n)=\int[1-x_h(u_l)]\,(c_h-c_l)d\left(F_l(u_l)^n\right). \tag{61}$$

Solving (60)–(61), we see that the volume of trade under the optimal mechanism between buyers and high-quality sellers with n offers is

$$x(h,n)=\int x_h(u_l)d\left(F_l(u_l)^n\right). \tag{62}$$

Using (62), similar algebra reveals that the transfers satisfy

$$t(i,n)=\int t_i(u_l)d\left(F_l(u_l)^n\right). \tag{63}$$

An immediate consequence of (62) and (63) is that buyers' utility coincides with what they receive in equilibrium, which proves the claim for $\phi_l>0$.

Consider next the case of $\phi_l<0$. We claim that equilibrium is constrained efficient if, and only if, $x_h(u_l)=1$ for all $u_l\in \text{Supp}(F_l)$. To see why, suppose that the equilibrium satisfies

$$\int x_h(u_l)d\left(F_l(u_l)^n\right)<1$$

for some $n\in\{1,2\}$. We will show that a perturbation of such an allocation is feasible and increases buyers' utility, i.e., the initial allocation cannot be constrained efficient. To do so, consider the mechanism

$$\begin{aligned}
t(l,n)&=V^s(l,n)\\
x(l,n)&=1\\
t(h,n)&=\int t_h(u_l)d\left(F_l(u_l)^n\right)\\
x(h,n)&=\int x_h(u_l)d\left(F_l(u_l)^n\right).
\end{aligned}$$

This mechanism satisfies the incentive and individual rationality constraints by construction. Now consider the following perturbation: for some n and $\epsilon>0$, let

$$\begin{aligned}
\hat{t}(l,n)&=t(l,n)+(c_h-c_l)\epsilon\\
\hat{x}(l,n)&=1\\
\hat{t}(h,n)&=t(h,n)+c_h\epsilon\\
\hat{x}(h,n)&=x(h,n)+\epsilon.
\end{aligned}$$

We argue that this perturbation remains feasible and strictly increases buyers' utilities. Note that incentive constraints are satisfied since

$$\hat{t}(l,n) = t(l,n) + (c_h - c_l)\epsilon \geqslant t(h,n) + (1-x(h,n))c_l + (c_h - c_l)\epsilon = \hat{t}(h,n) + (1-\hat{x}(h,n))c_l.$$

Moreover, this perturbation raises the payoff of low-quality sellers and leaves the payoff of high-quality sellers unchanged. Finally, buyers' payoffs rise since the net impact of this perturbation is given by

$$[\mu_h(\nu_h - c_h) - \mu_l(c_h - c_l)]\,\epsilon = -\phi_l\mu_l(c_h - c_l) > 0,$$

where the last inequality follows from $\phi_l < 0$.

The final step of the proof requires showing that a pooling equilibrium—with $x_h(u_l) = 1$ for all $u_l \in \mathrm{Supp}(F_l)$—is constrained efficient. To see why, note that $V^s(l,n) = V^s(h,n)$ for $n \in \{1,2\}$ in any incentive compatible mechanism with full trade. Since the sellers' participation constraint binds, and the total surplus generated by the constrained efficient allocation coincides with that in the equilibrium, the payoff to the buyers must coincide as well.

C General Trading Mechanisms

In our equilibrium construction, we assumed that buyers offer menus consisting of two contracts—one for high-quality sellers and one for low-quality sellers. In this section, we show that this assumption is without loss of generality. In particular, we consider a game where sellers can send arbitrary messages and buyers offer mechanisms that are deterministic and exclusive—but otherwise unrestricted—mapping the seller's message into potential terms of trade.[5] We prove that the distribution of trades in any equilibrium of this more general setting coincides with that of a game with two-point menus. We prove this within the context of our baseline model, where two buyers face a continuum of sellers.

Intuitively, this result essentially shows that it is impossible for a buyer to screen a seller based on her outside offer. To see why, note that screening is possible only when the payoffs from accepting a given contract differ across types. For example, a seller with a low-quality good gets less utility (compared to one with a high-quality good) from accepting a contract that requires her to retain a fraction of the good. However, sellers who differ only in their alternative offers get the *same* utility from accepting a contract; since trading is exclusive, once they accept the terms of a given contract, their outside offer is *irrelevant*. This feature rules out the ability to screen sellers along this dimension.

The proof proceeds in two steps. First, we map our environment into the general framework of Martimort and Stole (2002), hereafter MS. This allows us to apply their "delegation principle," which establishes that any equilibrium of a game with general mechanisms and messages can be achieved by a menu game. Second, we show that equilibrium menus have at most two contracts that are accepted by sellers in equilibrium. Together, these steps imply that a game where buyers offer 2-point menus induces the same equilibrium distribution of trades as a more general game with arbitrary mechanisms and communication.

Step 1. We begin by expressing payoffs and strategies using the notation of MS. A contract is defined by a quantity-transfer pair $d = (x, t)$. The seller's type is given by $\theta = (j, A)$, where $j \in \{l, h\}$ is the quality of her good and $A \subset \{1, 2\}$ is the set of buyers with whom she is matched. Given a pair of contracts offered by the two buyers, $d = \left(d^1, d^2\right)$, the payoff to a seller of type θ is

$$U(d;\theta) = \max_{i \in A} \quad t^i + \left(1 - x^i\right)c_j. \tag{64}$$

When a seller has access to both of the buyers and is indifferent between the contracts they offer, we assume she randomizes, with each buyer being chosen with equal probability. We denote the seller's

[5] In a deterministic mechanism, the mapping from the seller's message to an offer is a deterministic function. Note, however, that buyers can still randomize over different mechanisms.

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contract choice by $s^i(d;\theta)$, where $s^1(d;\theta)+s^2(d;\theta)=1$, so that the buyer's payoff can be written as

$$V^i(d;\theta)=\left(v_j x^i - t^i\right)s^i(d;\theta). \tag{65}$$

There is an unrestricted space of messages, denoted $\mathcal{M}$, available to each buyer-seller pair. The strategy space for buyers is the space of all deterministic communication mechanisms. Formally, such a mechanism consists of a mapping $\hat{d}^i:\mathcal{M}\to\mathcal{D}$ from messages to the set of all contracts $\mathcal{D}=[0,1]\times\mathbb{R}_+$. The set of such mechanisms is represented by $\Upsilon=(\mathcal{D})^{\mathcal{M}}$. Each buyer's strategy σ^i, then, is a distribution over the elements of Υ. A seller's strategy is a joint distribution over messages sent to each buyer with whom she is matched. The timing of the game is as follows. First, sellers draw their types. Second, each of the buyers simultaneously offers a mechanism to the sellers with whom they are matched. Third, each seller chooses a message to send to each of the buyers with whom she is matched. These choices then induce (potentially a pair of) contracts, with the resulting payoffs given by (64)–(65).

We can now apply the delegation principle from MS (Theorem 1). It states that the distribution of contracts and trades induced by any Perfect Bayesian Equilibrium in the game with mechanisms can be achieved by a game where buyers post menus of contracts and sellers choose their desired contract. Formally, a menu game is one in which each buyer's strategy is a distribution (possibly random) $\mu\in\Delta\left(2^{\mathcal{D}}\right)$ over all possible menus $z\subset\mathcal{D}$. Facing two menus, a seller of type j proceeds in two steps. First, she chooses a contract from each menu, which is described by a probability distribution $\chi_j(z_1,z_2;\theta)\in\Delta(z_1\times z_2)$ over pairs of contracts $d\in z_1\times z_2$. She then chooses one of the two contracts according to the functions $s^i(\cdot)$ described above.

Step 2. The second step, stated formally in the following result, shows that equilibrium menus cannot contain more than two "active" contracts, i.e., ones that are actually traded in equilibrium.

Proposition 2. *In any equilibrium of the menu game, any menu z has at most two contracts that are chosen by some seller type in equilibrium.*

Proof. Without loss of generality, consider an arbitrary menu z offered by buyer 1 with positive probability in equilibrium, and define $D_j(z)$ as the set of all contracts in that menu that are chosen by a type j seller with positive probability, i.e.,

$$D_j(z)=\{d^1\in z:\ \exists\, d^2\in z'\in \mathrm{Supp}(\mu):\left(d^1,d^2\right)\in\mathrm{Supp}(\chi_j(z,z')),s^1(d^1,d^2;(j,\cdot))>0\}.$$

We will show that $\left|D_j(z)\right|=1$ for $j\in\{l,h\}$. The strategy is to show that all elements in $D_j(z)$ must yield the same utility to type j sellers *and* the same payoffs to the buyer, i.e., for all $(x,t),(x',t')\in D_j(z)$, we must have

$$t+c_j(1-x)=t'+c_j\left(1-x'\right) \tag{66}$$
$$v_j x - t = v_j x' - t', \tag{67}$$

which implies $(x,t)=(x',t')$. It is easy to see that the two contracts must offer the same utility to the seller; otherwise she cannot choose both from the same menu with positive probability. To show that they must yield the same payoff to the buyer, consider the offer intended for the type l seller. Now, suppose that $(x,t),(x',t')\in D_l(z)$ with $v_l x - t > v_l x' - t'$. This inequality, combined with (66), implies

$$v_l\left(x-x'\right)>t-t'=c_l\left(x-x'\right)\Rightarrow x>x'.$$

As a result,

$$c_h\left(x-x'\right)>c_l\left(x-x'\right)=t-t'\Rightarrow t+c_h(1-x)>t'+c_h\left(1-x'\right).$$

This implies that $(x',t')\notin D_h(z)$. Hence, if we eliminate from z every contract in $D_l(z)$ except the one that delivers the maximum payoff to the buyer from type l sellers, the buyer's payoff strictly increases

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and the high type seller's choice is not altered. Therefore, if there is more than one element in $D_l(z)$, they must all yield the same profits.

Now suppose there exist $(x,t),(x',t') \in D_h(z)$ such that $v_h x - t > v_h x' - t'$. As before, this implies $x > x'$. We then have

$$t - t' = c_h\left(x - x'\right) > c_l\left(x - x'\right) \Rightarrow t + c_l\left(1 - x\right) > t' + c_l\left(1 - x'\right).$$

Hence, $(x',t') \notin D_l(z)$. Then, as with type l sellers, eliminating all contracts in $D_h(z)$ that deliver less than the maximum payoff to the buyer is a profitable deviation. This concludes the proof.

D Mass Point Equilibria: The Case of $\phi_l = 0$

Proposition 3. *Suppose $\phi_l = 0$. The unique equilibrium of the game is described by the pair of distribution functions, with $F_l(u_l)$ degenerate at v_l and $F_h(u_h)$ satisfying*

$$\left(1 - \pi + \pi F_h\left(u_h\right)\right)\mu_h \Pi_h\left(v_l, u_h\right) = \left(1 - \pi\right)\mu_l\left(v_l - c_l\right) \tag{68}$$

with $\text{Supp}(F_h) = [c_h, c_h + \pi\left(v_l - c_l\right)\left(v_h - c_h\right)/(v_h - c_l)]$.

Proof of Proposition 3. To show that the constructed distributions constitute an equilibrium, we show that there are no profitable deviations. In other words,

$$\forall\left(u_h', u_l'\right): \mu_h\left(1 - \pi + \pi F_l\left(u_l'\right)\right)\Pi_h\left(u_l', u_h'\right) + \mu_l\left(1 - \pi + \pi F_l\left(u_l'\right)\right)\left(v_l - u_l'\right) \leqslant \left(1 - \pi\right)\mu_l\left(v_l - c_l\right).$$

We consider two cases:

1. $u_h' > \max \text{Supp}\left(F_h\right) = \bar{u}_h$: In this case, when $u_l' > v_l$, the profit function is given by

$$\mu_h \Pi_h\left(u_l', u_h'\right) + \mu_l\left(v_l - u_l'\right).$$

   Since $\phi_l = 0$, this function is invariant to changes in u_h' and is strictly decreasing in u_l'. Therefore, its value must be less than its value evaluated at $(\bar{u}_h, v_l)$, which gives the equilibrium profits. When, $u_l' \leqslant v_l$, the profits are given by $\mu_h \Pi_h\left(u_l', u_h'\right)$, which is decreasing in u_h', and therefore

$$\mu_h \Pi_h\left(u_l', u_h'\right) + \mu_l\left(1 - \pi\right)\left(v_l - u_l'\right) < \mu_h \Pi_h\left(u_l', \bar{u}_h\right) + \mu_l\left(1 - \pi\right)\left(v_l - u_l'\right).$$

   Note that the right-hand side of the above inequality is a linear function of u_l', whose derivative is given by

$$\begin{aligned} \mu_h \frac{v_h - c_h}{v_l - c_l} - \mu_l\left(1 - \pi\right) &= \mu_h \frac{v_h - c_h}{c_h - c_l} - \mu_l + \mu_l \pi \\ &= -\mu_l \phi_l + \mu_l \pi = \mu_l \pi > 0. \end{aligned}$$

   Therefore, we must have that

$$\mu_h \Pi_h\left(u_l', \bar{u}_h\right) + \mu_l\left(1 - \pi\right)\left(v_l - u_l'\right) \leqslant \mu_h \Pi_h\left(v_l, \bar{u}_h\right) = \left(1 - \pi\right)\mu_l\left(v_l - c_l\right),$$

   where the last equality follows from (68).

2. $u_h' \in [c_h, \bar{u}_h]$. In this case, when $u_l' > v_l$, profits are given by

$$\begin{aligned} \mu_h\left(1 - \pi + \pi F_l\left(u_h'\right)\right)\Pi_h\left(u_l', u_h'\right) + \mu_l\left(v_l - u_l'\right) &\leqslant \mu_h\left(1 - \pi + \pi F_l\left(u_h'\right)\right)\Pi_h\left(v_l, u_h'\right) \\ &= \left(1 - \pi\right)\mu_l\left(v_l - c_l\right), \end{aligned}$$

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where the inequality is satisfied since $u_l' > \nu_l$ and the last equality follows from (68).

When $u_l' \leqslant \nu_l$, profits are given by

$$\mu_h \left(1-\pi+\pi F_l\left(u_h'\right)\right) \Pi_h\left(u_l', u_h'\right) + \mu_l (1-\pi)\left(\nu_l - u_l'\right).$$

This function is linear in u_l' and its derivative is given by

$$\begin{aligned} \mu_h \left(1-\pi+\pi F_l\left(u_h'\right)\right) \frac{\nu_h - c_h}{c_h - c_l} - \mu_l (1-\pi) &= (1-\pi)\left(\mu_h \frac{\nu_h - c_h}{c_h - c_l} - \mu_l\right) + \pi F_l\left(u_h'\right) \frac{\nu_h - c_h}{c_h - c_l} \\ &= \pi F_l\left(u_h'\right) \frac{\nu_h - c_h}{c_h - c_l} \geqslant 0. \end{aligned}$$

Therefore, it is maximized at $u_l' = \nu_l$. This establishes that there are no profitable deviations.

To conclude the proof, we show that the equilibrium constructed is the unique equilibrium when $\phi_l = 0$.

In order to show uniqueness of the equilibrium, it is sufficient to show that, in any equilibrium, F_l must be degenerate at ν_l. When F_l is degenerate at ν_l, from Lemmas 1 and 4, we know that F_h must be continuous and strictly increasing, and therefore it must satisfy (68).

Suppose that $u_l \neq \nu_l$ exists that belongs to the support of F_l. Then the proof of Lemma 5 can be used to show that for values of $u_l \neq \nu_l$, F_l must have no flat and mass points and, consequently, equilibrium must exhibit the strict rank-preserving (SRP) property. Now consider any menu for which $u_l < \nu_l$ and a deviation that increases the value of u_l by a small amount. In this case, F_l is differentiable and we can write the change in profits from such a deviation as

$$\begin{aligned} &\mu_l \pi f_l^+(u_l)(\nu_l - u_l) - \mu_l(1-\pi+\pi F_l(u_l)) + \mu_h \frac{\nu_h - c_h}{c_h - c_l}(1-\pi+\pi F_h(u_h)) = \\ &\mu_l \pi f_l^+(u_l)(\nu_l - u_l) - \mu_l \phi_l (1-\pi+\pi F_l(u_l)) > 0 \end{aligned}$$

where in the above f_l^+ is the right derivative of F_l and we have used SRP. The above implies that increasing u_l must be a profitable deviation, which proves the contradiction. The case with $u_l > \nu_l$ is ruled out in a similar fashion. This concludes the proof. ■

E Additional Extensions and Robustness

In this section, we examine a few additional extensions of our framework, both to ensure the robustness of our results and to demonstrate that our framework is amenable to more applied work. First, we relax our assumption of linear utility to analyze the canonical model of insurance under private information. Second, we allow the degree of competition to differ across sellers of different quality. Third, we incorporate additional dimensions of heterogeneity, including horizontal and vertical differentiation. Lastly, we consider the case of $N > 2$ types of sellers. All proofs are in Section E.5.

E.1 A Model of Insurance

To start, we analyze a canonical model of insurance under private information, along the lines of Rothschild and Stiglitz (1976), and show that our main results—in particular, the structure of equilibrium menus and the nonmonotonicity of welfare with respect to the degree of competition—extend beyond the linear, transferable utility environment.

A unit measure of agents with strictly increasing, strictly concave utility functions $w(c)$ face idiosyncratic income risk.[6] Their income in normal times is y, but they also face the risk of an "accident,"

[6]Note that, in this application, the "buyers" of insurance are the ones with private information. To avoid confusion, we

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which reduces their income by d. The accident is observable and contractible, but the probability of its occurrence, denoted $\theta_j,\ \ j \in \{b, g\}$, is private information. A fraction μ_b of agents are of type b and face a higher risk of accident than type g agents, i.e., $\theta_b > \theta_g$. Principals (the insurance providers) are risk-neutral, which implies that gains from trade are strictly positive for both types. The competitive structure is exactly the same as our baseline model: a fraction $1-\pi$ of agents receive one offer and the remainder receive two.

A contract consists of a premium and a transfer to the agent in the event of an accident. Since trading is exclusive and the accident is observable, we can also think of the contract as directly offering a utility level in the normal and accident states. As before, we consider menus with two contracts, one for each type, i.e., $\mathbf{z} = \left(u_b^n, u_b^a\right), \left(u_g^n, u_g^a\right)$ such that incentive and participation constraints are satisfied for $j \in \{b, g\}$:

$$\begin{aligned} \left(IC_j\right): \quad & \theta_j u_j^a + \left(1-\theta_j\right) u_j^n \geqslant \theta_j u_{-j}^a + \left(1-\theta_j\right) u_{-j}^n, \\ \left(PC_j\right): \quad & \theta_j u_j^a + \left(1-\theta_j\right) u_j^n \geqslant \theta_j w\left(y-d\right) + \left(1-\theta_j\right) w\left(y\right). \end{aligned}$$

To solve for the equilibrium, we follow the same steps as in Section 4. The first step is to obtain the utility representation. It is straightforward to prove that, in all equilibrium menus, type b agents are fully insured and $\left(IC_b\right)$ binds. This allows us to summarize equilibrium menus with a pair of expected utilities, (u_b, u_g), and allocations given by the solution to the following system of equations:

$$u_b = u_b^a = u_b^n, \qquad u_b = \theta_b u_g^a + (1-\theta_b)\, u_g^n, \qquad u_g = \theta_g u_g^a + (1-\theta_g)\, u_g^n. \tag{69}$$

In a separating menu, the principal offers type g agents less than full insurance: $u_g^a < u_g^n$ such that $\left(IC_b\right)$ binds. Define $C\left(u\right) \equiv w^{-1}\left(u\right)$ to be the principal's cost of providing a utility level u. Note that $C'\left(u\right), C''\left(u\right) > 0$. Then, the objective of the principal is described by (8), where the type-specific profit functions satisfy

$$\begin{aligned} \Pi_b\left(u_b, u_g\right) &= y - \theta_b d - C\left(u_b\right), \\ \Pi_g\left(u_b, u_g\right) &= y - \theta_g d - \theta_g C\left(u_g^a\right) - \left(1-\theta_g\right) C\left(u_g^n\right). \end{aligned}$$

Since w is strictly increasing and concave, we can show that

$$\frac{d\Pi_g\left(u_b, u_g\right)}{du_b} > 0, \qquad \text{and} \qquad \frac{d\Pi_g\left(u_b, u_g\right)}{du_g\, du_b} > 0\,.$$

The first inequality shows the effect of incentives: more surplus to type b agents relaxes their incentive constraint, allowing the principal to earn higher profits from type g agents. The second inequality shows that the marginal benefit of increasing the utility of type g agents rises with the utility offered to type b agents, implying the strict supermodularity of the profit function. In other words, the complementarity that was at the heart of the strict rank-preserving property in the linear model is present in this version as well. Using this property, we can extend the arguments in Proposition 1, implying that the marginal distributions $F_j,\ j \in \{b, g\}$ do not have any flat portions or mass points. Hence, Theorem 1 applies—equilibria are strictly rank-preserving—and can therefore be described by a distribution over utilities to type b agents, $F_b(u_b)$, and a strictly increasing function $U_g(u_b)$. In Appendix E.5.1, we use the methods from Section 4 to derive the system of differential equations that characterize these functions.

Next, we consider the implications of competition for welfare. For brevity, we restrict attention to the region where all menus are separating and do not involve cross-subsidization. In this case, the consumption of type g agents necessarily varies with the state; this imperfect insurance is the analogue of distortions in the quantity traded in the baseline model. The associated resource costs are thus a natural measure of the efficiency losses (relative to a full information benchmark) in this setting. For a

switch to a principal-agent description.

menu offering u_b to type b agents, this loss is given by

$$L(u_b) \quad = \quad C\left(U_g(u_b)\right) - \left[\theta_g C\left(U_g^a(u_b)\right) + (1-\theta_g)\, C\left(U_g^n(u_b)\right)\right] \,, \tag{70}$$

where U_g, U_g^a, and U_g^n are equilibrium policy functions. Average losses in the economy are then

$$\mathcal{L}(\pi) \quad \equiv \quad (1-\pi)\int L(u_b)dF_b(u_b,\pi) + \pi\int L(u_b)dF_b(u_b,\pi)^2 \,. \tag{71}$$

In Appendix E.5.1, we show, using a numerical example, that L is U-shaped in u_b, which then implies that $\mathcal{L}(\pi)$ is minimized at an interior value of π. Thus, in markets for insurance, increasing competition among providers can be detrimental for welfare.

E.2 Differential Competition Across Types

In our baseline model, we assume that the probability a seller receives one or two offers is the same for both types. In this subsection, we relax this assumption and allow π to vary across types, so that the probability a type j seller is captive is given by $1-\pi_j$. We will show that both the structure of the equilibrium and its normative properties remain largely unchanged, with the caveat that, for some parameter values, the equilibrium distribution has mass points. For brevity, we restrict attention to the $\phi_l > 0$ case, where all equilibrium menus are separating and cross-subsidization does not occur.

When $\pi_h > \pi_l$, the results in Proposition 1 go through unchanged, and thus the distribution functions F_l and F_h have continuous support and no mass points. This implies that the equilibrium satisfies the strict rank-preserving property and all menus attract the same fraction of noncaptive sellers. When $\pi_l > \pi_h$, both distributions still have continuous supports, but F_l has a mass point if π_l is sufficiently large. The following proposition fully characterizes the unique equilibrium for both cases.

Proposition 4. *If* $\frac{1-\pi_l}{1-\pi_h} < 1-\phi_l$*, then the unique equilibrium* F_l *has full mass at* v_l *and* F_h *is characterized by*

$$(1-\pi_h+\pi_h F_h(u_h))\,\Pi_h\left(v_l,u_h\right) = (1-\pi_h)\,\Pi_h\left(v_l,c_h\right). \tag{72}$$

If $\frac{1-\pi_l}{1-\pi_h} \geqslant 1-\phi_l$*, then the unique equilibrium* F_l *satisfies*

$$\frac{\pi_l f_l(u_l)}{1-\pi_l+\pi_l F_l(u_l)}\Pi_l(u_l) \quad = \quad 1 - \frac{1-\pi_h+\pi_h F_l(u_l)}{1-\pi_l+\pi_l F_l(u_l)}\left(\frac{\mu_h}{\mu_l}\right)\frac{v_h-c_h}{c_h-c_l} \tag{73}$$

and U_h *is determined by the equal profit condition.*

Equation (73) is similar in structure to (16). The key difference is that the right-hand side, which again measures the (net) marginal cost of providing a unit of surplus to the low type, has an additional term that adjusts for the differential probability that an offer is *accepted* by high types relative to low types. Naturally, this probability is small (i.e., the cost is large) when u_l is small and π_h is large.

The construction of equilibrium follows the strategy in Section 4. The ordinary differential equation in (73), with the boundary condition $F_l(c_l) = 0$, can be solved for F_l. Given F_l, the equal profit condition pins down U_h. The properties of the equilibrium—both positive and normative—are also similar to the baseline model. In particular, x_h is nonmonotonic in u_l which, as before, has interesting implications for the relationship between welfare and competition.

Figure 4 illustrates the effects of varying competition for each type separately. The left panel varies π_h, holding π_l fixed, and shows that more competition for high-quality sellers always reduces welfare; intuitively, more surplus to high-quality sellers tightens the incentive constraints and reduces trade. The right panel varies π_l, holding π_h fixed, which has two effects (exactly as in section 5.2). First, it increases surplus to low-quality sellers, which relaxes incentive constraints and increases trade with high-quality sellers. Second, it makes low-quality sellers relatively less attractive to buyers, inducing them to compete

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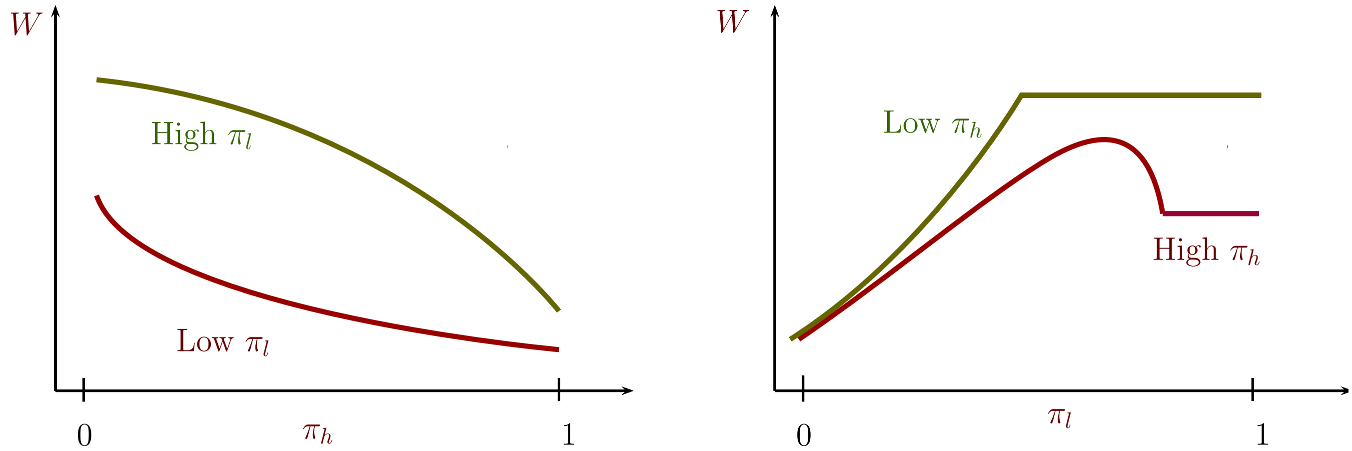

Figure 4: The effect of varying competition on welfare for high- (left panel) and low-quality (right panel) sellers

more aggressively for the high-quality seller, thereby reducing trade. These two competing forces lead to a nonmonotonic relationship between π_l and welfare, provided π_h is sufficiently high.[7]

E.3 Differentiation and Multidimensional Heterogeneity

In this section, in order to enhance the applicability of our framework to applied work, we introduce various types of additional heterogeneity: across buyers, across contracts, and across sellers. In various ways, these generalizations break the stark relationship between a seller's type, the offer she accepts, and the rank of that offer within the distribution of all offers. The cost of these generalizations is some degree of tractability, though we argue that, in most cases, the properties and characterization of equilibria are very similar to the baseline framework. For brevity, we restrict attention to the region of the parameter space where almost all equilibrium menus are separating and not cross-subsidizing.

Horizontal differentiation across buyers. Consider first the possibility that buyers are horizontally differentiated. Specifically, as in the discrete choice model of McFadden (1974), we assume that the payoff to a seller of type i from a contract (x, t) offered by buyer k is

$$u_{ik} = (1 - x)\, c_i + t + \epsilon_k = u_i + \epsilon_k ,$$

where ϵ_k is a buyer-specific preference shock drawn from a continuous distribution H with support $[\underline{\epsilon}, \bar{\epsilon}]$. Note that ϵ is the same for both seller types, so it has no effect on the incentive constraints. Hence, we may once again represent each equilibrium menu by a utility pair (u_l, u_h). A captive seller accepts this menu if u_{ik} is greater than her outside option, c_i, which occurs with probability

$$\tilde{F}_i^c\,(u_i) = \int_{c_i - u_i}^{\bar{\epsilon}} dH(\epsilon) = 1 - H\,(c_i - u_i)\ . \tag{74}$$

A noncaptive seller of type i accepts this menu if $u_i + \epsilon > \max(u_i' + \epsilon', c_i)$, which occurs with probability

$$\tilde{F}_i^{nc}\,(u_i) = \int_{\underline{u}_i}^{\bar{u}_i} \int_{c_i - u_i}^{\bar{\epsilon}} \left(\int_{\bar{\epsilon}}^{u_i + \epsilon - u_i'} dH\left(\epsilon'\right) \right) \quad dH\left(\epsilon\right) \quad dF_i\left(u_i'\right), \tag{75}$$

[7] When π_h is low, we enter the region with mass points before the second (negative) effect begins to dominate. Since a mass point equilibrium puts full mass at v_l, increasing π_l beyond this point has no effect on welfare.

where F_i is the marginal distribution of utilities offered to type i sellers in equilibrium. Setting $M_i(u_i) = (1-\pi)\tilde{F}_i^c(u_i') + \pi\tilde{F}_i^{nc}(u_i')$, we can write the buyer's problem as

$$\max_{u_l',\,u_h'} \quad \sum_{i\in\{l,h\}} M_i(u_i')\,\Pi_i(u_l', u_h')\,. \tag{76}$$

In a separating equilibrium, optimality with respect to u_l requires

$$\frac{m_l(u_l)}{M_l(u_l)}(v_l - u_l) = \phi_l. \tag{77}$$

In other words, the link between the trading probability and the utility offered to the low-quality seller is exactly the same as in our baseline framework, and all of our results go through with respect to the key equilibrium objects M_l and M_h. The only caveat is that recovering the underlying distribution of offers F_l and F_h, which are informative about prices and allocations, typically requires numerical methods.[8]

Horizontal differentiation across contracts. The extension above allows for the possibility that a seller accepts a contract from the "wrong" buyer, i.e., accepts u_i even though a contract $u_i' > u_i$ was available. In this section, we allow for the possibility that a seller accepts the "wrong" contract within a menu, i.e., accepts u_{-i} even though her type is i. In particular, suppose that a fraction δ of low-quality sellers accept the contract intended for a high-quality seller. It is possible to microfound this as a form of "tremble," or as arising from other unmodeled contract features that cause some low-quality sellers to prefer the contract with lower quantity and higher price.[9] For example, the high-price contract might carry other benefits, such as better customer service, that are valued by some low-quality sellers (but not others).

Let $\tilde{v}_h \equiv \frac{\mu_h v_h + \mu_l \delta v_l}{\mu_h + \mu_l \delta}$ be the average value (to the buyer) of goods held by agents who take the contract intended for the high type. We assume that δ is sufficiently small so that $\tilde{v}_h > c_h$. The expected profits of the buyer, conditional on trade, are then given by $\tilde{\Pi}_h(u_l, u_h) = \tilde{v}_h - \left(\frac{\tilde{v}_h - c_l}{c_h - c_l}\right)u_h + \left(\frac{\tilde{v}_h - c_h}{c_h - c_l}\right)u_l$. As in our baseline model, the FOC for u_l and the equal profit condition pin down F_l and U_h:

$$\begin{aligned}
\frac{\pi f_l(u_l)}{1-\pi+\pi F_l(u_l)}(v_l - u_l) &= 1 - \frac{\mu_h + \mu_l\delta}{\mu_l(1-\delta)}\left(\frac{\tilde{v}_h - c_h}{c_h - c_l}\right) \equiv \tilde{\phi}_l, &(78)\\
(1-\pi)\mu_l\delta(v_l - c_l) &= (1-\pi+\pi F_l(u_l))\left[\mu_l(1-\delta)(v_l-u_l) + (\mu_h+\mu_l\delta)\tilde{\Pi}_h(u_l,u_h)\right]. &(79)
\end{aligned}$$

Note that these equations are very similar to (16)–(17), with $\tilde{\Pi}_h$ and $\tilde{\phi}_l$ replacing Π_h and ϕ_l. Accordingly, the characterization and other results in the preceding sections directly extend.

Vertical differentiation across buyers. Suppose now that sellers attach a higher value to trading with certain buyers, i.e., that the utility of a type i seller from accepting a contract (x, t) from buyer $k \in \{1, 2\}$ is given by $c_i(1-x) + t + B^k$, where $B^1 \equiv B > 0$ and B^2 is normalized to zero.[10] This implies that the cost of delivering utility to sellers is lower for buyer 1 or, equivalently, his profits are higher than those of buyer 2, i.e., $\Pi_i^1(u_l, u_h) = \Pi_i^2(u_l, u_h) + B$. Not surprisingly, in this environment, the equilibrium distribution of menus is also asymmetric. Let $F_i^k(u_i)$, $k \in \{1, 2\}$ denote the marginal distribution of utilities offered by buyer k to type j sellers. In Appendix E.5.3, we characterize an equilibrium in which

[8]The differential equation in (77), along with the equal profit condition and the system of integral equations in (74)–(75) must be solved jointly for F_i, and this system is only analytically tractable under special assumptions on the distribution H.

[9]For simplicity, we make two additional assumptions. First, a captive low-quality seller still chooses the more attractive menu, even when she takes the contract intended for the high-quality seller. Second, we assume that the buyer does not (or cannot) try to use contract terms to separate out these low-quality sellers.

[10]Equivalently, and more consistent with our earlier interpretation, one could imagine a measure of buyers, with a fraction of each type $k \in \{1, 2\}$. The simplification here implies that a noncaptive seller will always have one offer from a type 1 buyer and one from a type 2 buyer, though this could be relaxed.

these distributions satisfy the strict rank-preserving property, except at the lower bound of the support, where F_i^2 has a mass point.[11]

Multidimensional seller heterogeneity. Finally, our baseline framework posits a tight connection between the valuations of the seller and the buyer. While this is a natural assumption when sellers are heterogeneous along a single dimension—asset quality—it is also natural to consider the case in which sellers have heterogenenous preferences as well. A simple way to incorporate this additional heterogeneity into our analysis is to assume that a seller's type is a tuple $(c, \tilde{v})$, with $c \in \{c_h, c_l\}$ denoting the seller's valuation for her asset and $\tilde{v} \in \{\tilde{v}_h, \tilde{v}_l\}$ denoting the buyer's valuation. This allows for the possibility that some high- (low-) quality assets are held by sellers who, for idiosyncratic reasons, have a low (high) valuation for them. In an asset market interpretation, for example, this could arise from heterogeneity in discount rates or liquidity needs. Let μ_{ij} denote the proportion of sellers of type $(c_i, \tilde{v}_j)$. We can show that it is not possible for buyers to separate sellers with the same c but different $\tilde{v}$'s. Let $\mu_i = \sum_j \mu_{ij}$ denote the fraction of sellers with valuation c_i, $i \in \{h, l\}$ and $\nu_i = \frac{\sum_j \mu_{ij}\tilde{v}_j}{\mu_i}$ denote the average value (to the buyer) of the assets held by sellers of type i. Assuming that gains from trade are positive, so that $c_i < \nu_i$, it is easy to see that our analysis of the baseline model goes through exactly. In other words, additional preference heterogeneity changes the interpretation of buyer values in our baseline model, but otherwise leaves the analysis unchanged.

E.4 The Model with Many Types

We now extend our analysis to the case with an arbitrary, finite number of seller types. We focus our attention on equilibria where all offers are separating menus. We do so for two reasons. First, in the case of $N = 2$, this region yields some of the most interesting results—such as the nonmonotonicity of welfare in π—and we want to confirm that these results are true in a more general setting. Second, in the equilibrium with all separating menus, the monotonicity constraints are slack ($x_i < x_{i+1}$), which is the most commonly studied case in the mechanism design literature.[12] We first provide a method for constructing such a separating equilibrium, and then use the constructed equilibrium to demonstrate that the welfare implications from the model with two types extend to the general case of $N > 2$.

Suppose there are $N \geqslant 2$ types, with buyers and sellers deriving utility ν_i and c_i, respectively, per unit from a good of type $i \in \mathcal{N} \equiv \{1, ..., N\}$. The types are ordered so that $\nu_1 < \nu_2 < ... < \nu_N$ and $c_1 < c_2 < ... < c_N$, and there are gains from trading all types of goods, i.e., $\nu_i > c_i$ for all $i \in \mathcal{N}$. The distribution of types is summarized by the vector $(\mu_1, \ldots, \mu_N)$, with $\sum_{i \in \mathcal{N}} \mu_i = 1$. As in our benchmark model, sellers (of all types) are privately informed about the quality of their good and receive two offers with probability π and one offer with probability $1 - \pi$.

Equilibrium Properties. The definition of strategies and a (symmetric) equilibrium are identical to those in the model with two types, and hence we omit them for brevity. We begin our analysis, in Lemma 14 below, by establishing that buyers' offers never distort the quantity traded with the lowest type of seller, and that local incentive constraints always bind "upward," i.e., equilibrium offers always leave a type i seller indifferent between his contract and the one intended for type $i + 1$. As a result, a buyer's offer can again be summarized by the indirect utilities it delivers to each type $i \in \mathcal{N}$.

Lemma 14. *For almost all equilibrium menus:*

1. *There is full trade with the lowest type, so that $x_1 = 1$, and the local incentive constraints are binding upward, so that*

$$t_i + c_i (1 - x_i) = t_{i+1} + c_i (1 - x_{i+1}) \text{ for all } i = 1, 2.....N - 1;$$

[11]Our analysis requires one additional assumption: a seller who is indifferent between two menus chooses the one offered by buyer 1. The resulting system of differential equations can be solved numerically to obtain the equilibrium distributions.

[12]See, e.g., Fudenberg and Tirole (1991).

2. *Each menu can be summarized by a utility vector* $\mathbf{u} = (u_1, \cdots, u_N)$ *with* $u_i \geqslant c_i \; \forall \; i$ *and*

$$1 \geqslant \frac{u_N - u_{N-1}}{c_N - c_{N-1}} \geqslant \cdots \geqslant \frac{u_2 - u_1}{c_2 - c_1} \geqslant 0, \tag{80}$$

with the corresponding quantities and transfers given by

$$\begin{aligned} x_1 &= 1, \quad x_i = 1 - \frac{u_i - u_{i-1}}{c_i - c_{i-1}}, \quad i = 2, 3.....N \\ t_1 &= u_1, \quad t_i = u_i - \frac{c_i}{c_i - c_{i-1}} \left(u_i - u_{i-1}\right), \quad i = 2, 3.....N. \end{aligned} \tag{81}$$

This proof of Lemma 14 is a direct extension of the proof of Lemma 1, and hence it is omitted for brevity. Given the results, we can recast each buyer's problem in terms of the utility vector $\mathbf{u}$. In particular, given a family of marginal distributions $F_i(u_i)$ for $i \in \mathcal{N}$, each buyer chooses a vector $\mathbf{u}$ to solve

$$\max_{u_i \geqslant c_i} \sum_{i=1}^{N} \mu_i \left(1 - \pi + \pi F_i\left(u_i\right)\right) \Pi_i\left(u_{i-1}, u_i\right) \tag{82}$$

subject to the monotonicity constraints in (80), where (in a slight abuse of notation) profits per trade with a seller of quality i are given by

$$\begin{aligned} \Pi_1\left(u_1\right) &= v_1 - u_1, \\ \Pi_i\left(u_{i-1}, u_i\right) &= v_i - \frac{v_i - c_{i-1}}{c_i - c_{i-1}} u_i + \frac{v_i - c_i}{c_i - c_{i-1}} u_{i-1}, \qquad \text{for all } i = 2.....N. \end{aligned} \tag{83}$$

The program in (82) resembles a standard mechanism design problem, where the binding incentive constraints are substituted into the profit functions in (83). The monotonicity constraints in (80) are necessary to ensure that local incentive compatibility implies global incentive compatibility.

We now formally define a separating equilibrium, provide a characterization and a method for constructing such equilibria, and then use numerical examples to study their normative properties.

Definition 1. *An equilibrium is separating if the utility vector* $\mathbf{u}$ *associated with any equilibrium menu solves the relaxed problem of maximizing the objective in* (82) *ignoring the monotonicity constraints in* (80).

As a first step, in the conjectured equilibrium, one can use an induction argument to extend Proposition 1, establishing that all the distributions $F_i\left(u_i\right)$ are continuous with connected support. Since the profit function is strictly supermodular, any separating equilibrium must satisfy the strict rank-preserving property. The following proposition summarizes.

Proposition 5. *If* $\phi_1 = 1 - \frac{\mu_2}{\mu_1}\frac{v_2 - c_1}{c_2 - c_1} \neq 0$*, then, in any symmetric separating equilibrium,*

1. *For all* $i \in \mathcal{N}$, $F_i\left(\cdot\right)$ *has a connected support and is continuous.*

2. *There exists a sequence of strictly increasing real-valued functions* $\{U_i\left(u_1\right)\}_{i=2}^{N}$ *such that the utility vector associated with any equilibrium menu* $\mathbf{z}$ *satisfies:*

$$\mathbf{u}\left(\mathbf{z}\right) = \left(u_1\left(\mathbf{z}\right), U_2\left(u_1\left(\mathbf{z}\right)\right), U_3\left(u_1\left(\mathbf{z}\right)\right), \cdots, U_N\left(u_1\left(\mathbf{z}\right)\right)\right). \tag{84}$$

As in the model with two types, Proposition 5 greatly simplifies the construction of separating equilibria: it implies that we only need to characterize the distribution of offers to the lowest type,

$F_1(u_1)$, together with the sequence of functions $\{U_i(u_1)\}_{i=2}^N$.[13] The equilibrium distribution of utilities can then be derived from the fact that all types have the same ranking across equilibrium menus, i.e., $F_i(U_i(u_1)) = F_1(u_1)$ for all $i = 2, ..., N$.

Equilibrium construction. We now illustrate how to construct a separating equilibrium. Differentiability of the profit function in (82) implies that any separating equilibrium must satisfy

$$\frac{\pi f_i(U_i(u_1))}{1-\pi+\pi F_i(U_i(u_1))}\Pi_1(u_1) = \phi_i \tag{85}$$

$$\frac{\pi f_i(U_i(u_1))}{1-\pi+\pi F_i(U_i(u_1))}\Pi_i(U_{i-1}(u_1), U_i(u_1)) = \phi_i \quad \text{for all } i = 2, ..., N, \tag{86}$$

where ϕ_i, the marginal cost of increasing the utility of a seller of type i, is given by

$$\begin{aligned}
\phi_1 &= 1 - \frac{\mu_2}{\mu_1}\frac{v_2 - c_2}{c_2 - c_1} \\
\phi_i &= \frac{v_i - c_{i-1}}{c_i - c_{i-1}} - \frac{\mu_{i+1}}{\mu_i}\frac{v_{i+1} - c_{i+1}}{c_{i+1} - c_i}, \qquad \text{for all } i = 2, \cdots, N-1 \\
\phi_N &= \frac{v_N - c_{N-1}}{c_N - c_{N-1}}.
\end{aligned}$$

Equation (85) implies that F_1 must satisfy

$$\frac{\pi f_1(u_1)}{1-\pi+\pi F_1(u_1)} = \frac{\phi_1}{v_1 - u_1}. \tag{87}$$

Since the strict rank-preserving property implies that each U_i must satisfy $F_i(U_i(u_1)) = F_1(u_1)$, it must be the case that $U_i'(u_1) f_1(U_i(u_1)) = f_1(u_1)$. Substituting this result into (86) implies that the equilibrium functions U_i must satisfy the set of differential equations:

$$U_i'(u_1) = \frac{\phi_1}{\phi_i}\frac{\Pi_i(U_{i-1}(u_1), U_i(u_1))}{v_1 - u_1} \qquad \text{for all } i = 2, \cdots, N. \tag{88}$$

The system of differential equations (87) and (88) are ordinary first-order differential equations; to complete the characterization, we need only provide the appropriate boundary conditions. As in the two-type model, these conditions depend critically on the marginal costs, $(\phi_1, \ldots, \phi_N)$, and are closely tied to the outcome under monopsony. The following result shows that the solution to a monopsonist's problem can be represented in the form of a threshold type.

Lemma 15. *Let J denote the largest integer $i \in \{1, 2...N\}$ such that*

$$\sum_{i=1}^{J-1} \mu_i \phi_i < 0, \tag{89}$$

with $J = 1$ if $\sum_{i=1}^{k} \mu_i \phi_i > 0$ for all $k \in \{1, 2....N\}$. The solution to a monopsonist's problem is to set $u_i = c_J$ for $i \leqslant J$ and $u_i = c_i$ for $i > J$.

Intuitively, the accumulated marginal cost of trading with the first J types is negative ($\sum_{i=1}^{J-1} \mu_i \phi_i < 0$), so they are pooled. In contrast, for the remaining types, the information rents outweigh the potential

[13] This proposition relies on the assumption that the marginal cost of transfers to the lowest type net of any benefits arising from binding incentive constraints, ϕ_1, is nonzero. As in the two-type case, this assumption is required to show that equilibrium distributions do not have mass points.

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gains, so the monopsonist chooses not to trade with them.[14] The next result links this threshold J to the best and worst menu when $\pi > 0$.

Lemma 16. *Let J be as defined in Lemma 15. Then, in any equilibrium, the best menu has $u_i = u_J$ for $i < J$, and the worst menu has $u_i = c_i$ for all $i \geqslant J$.*

To see the intuition, note that the best menu trades with probability 1, i.e., attracts all captive and noncaptive sellers. Therefore, it cannot be profitable for that menu to separate types that a monopsonist finds profitable to pool; if $u_i < u_J$ for some $i < J$, then increasing u_i has no effect on the probability or composition of trades but yields strictly higher profits (because the effective marginal cost of increasing u_i is negative). Similarly, it cannot be profitable for the worst menu to give any surplus to the types that the monopsonist finds optimal to shut out completely; if such a menu offers more than c_i to any type $i > J$, the buyer can raise her profits simply by lowering that utility.

The system of differential equations (87)-(88), along with the boundary conditions described in Lemma 16, describe necessary conditions for any separating equilibrium. By the Picard-Lindelöf theorem, it has a unique solution. In Appendix E.5.4, we provide analytical expressions for this solution. To ensure that this solution is an equilibrium, one need only verify that the monotonicity constraints (80) are satisfied for every $u_1 \in \text{Supp}(F_1)$.

Finally, we solve two numerical examples using the method described above. The two cases both have $N = 4$, but differ in the marginal cost vector, $(\phi_1, ...\phi_N)$.[15] In the first case, $J = 1$, so the monopsonist only trades with the lowest type. In the second case, $J = 2$. We use these cases to demonstrate the robustness of the welfare results in section 5.2. In Figure 5, we plot expected trade for types 2 through 4 (recall that $x_1 = 1$ always) as a function of π. They show a nonmonotonic relationship between expected trade and competition. In the first case (left panel), in which the monopsonist only trades with type 1, trade by all three types is hump-shaped. This is analogous to the case with $\phi_l > 0$ in the two-type model. In the second case (right panel), however, trade with one of the types (type 2) is monotonically decreasing in π. This is similar to the case with $\phi_l < 0$ in the two-type model. In both cases, these patterns imply that ex-ante welfare is maximized at $\pi < 1$.

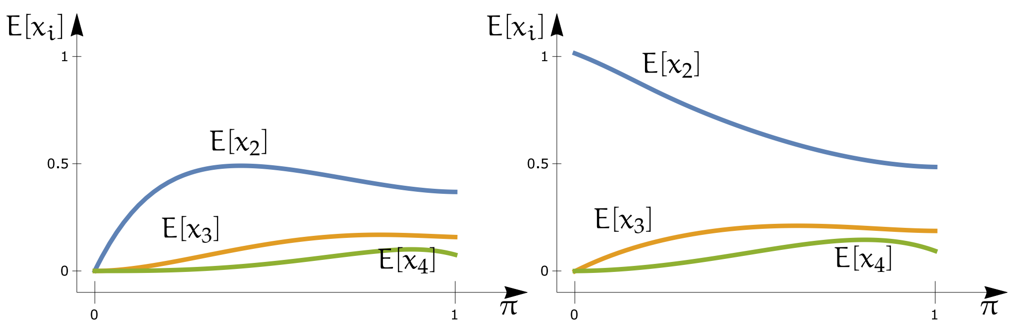


Figure 5: Expected trade and competition when $N = 4$ and $J = 1$ (left panel) or $J = 2$ (right panel).

E.5 Proofs

E.5.1 Construction of Equilibrium for the Insurance Model

The construction of equilibrium follows the logic of Section 4. For brevity, we focus on the region of the parameter space where all equilibrium menus are separating and involve no cross-subsidization.

[14] For brevity, we ignore the non-generic case in which the inequality in (89) is satisfied with equality.

[15] For both cases, we assume a uniform distribution $\mu_i = 0.25$ for all i, with valuations $c_i = 1, 2, 3, 4$ and $v_i = c_i\delta + 0.5$. In case 1, $\delta = 1.2$ and in case 2, $\delta = 1.3$. In each case, we solve the system (87)-(88) and verify that the monotonicity constraints are satisfied.

This obtains when the fraction of type-b agents, μ_b, is sufficiently large. The optimality conditions with respect to u_b and u_g in this case are

$$\frac{\pi f_b(u_b)}{1-\pi+\pi F_b(u_b)}\Pi_b(u_b) = C'(u_b) - \frac{\mu_g}{\mu_b}\left[\frac{\theta_g(1-\theta_g)}{\theta_b-\theta_g}C'(u_g^n) - \frac{\theta_g(1-\theta_g)}{\theta_b-\theta_g}C'(u_g^a)\right] \tag{90}$$

$$\frac{\pi f_g(u_g)}{1-\pi+\pi F_g(u_g)}\Pi_g(u_b,u_g) = \frac{(1-\theta_g)\theta_b}{\theta_b-\theta_g}C'(u_g^n) - \frac{\theta_g(1-\theta_b)}{\theta_b-\theta_g}C'(u_g^a). \tag{91}$$

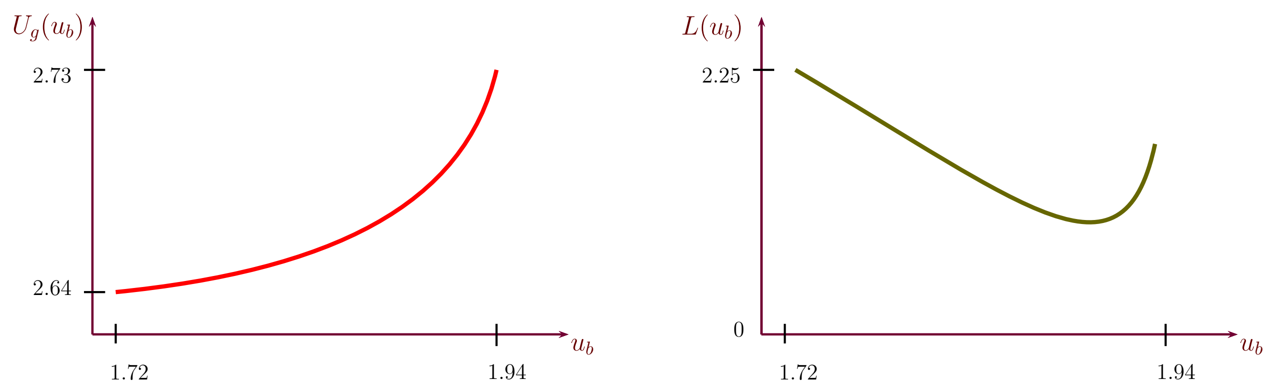
These two differential equations, along with the boundary conditions $F_j(\underline{u}_j) = 0$ with $\underline{u}_j \equiv \theta_j w(y-d) + (1-\theta_j)w(y)$, characterize the equilibrium. Note that these are similar in structure to (16), except that the marginal cost of delivering utility varies with the level of utility (this was constant in the linear model). To solve this system, we make use of the SRP relationship, $F_b(u_b) = F_g(U_g(u_b))$, which implies $f_b(u_b) = f_g(U_g(u_b))U_g'(u_b)$. Dividing the first differential equation by the second and using the SRP identities, we obtain

$$\frac{\Pi_b(u_b)U_g'(u_b)}{\Pi_g(u_b,U_g(u_b))} = \frac{C'(u_b) - \frac{\mu_g}{\mu_b}\left[\frac{\theta_g(1-\theta_g)}{\theta_b-\theta_g}C'(u_g^n) - \frac{\theta_g(1-\theta_g)}{\theta_b-\theta_g}C'(u_g^a)\right]}{\frac{(1-\theta_g)\theta_b}{\theta_b-\theta_g}C'(u_g^n) - \frac{\theta_g(1-\theta_b)}{\theta_b-\theta_g}C'(u_g^a)}, \tag{92}$$

where u_g^n and u_g^a are related to u_b and U_g through (69). Equation (92) is thus an ordinary differential equation in U_g, along with the boundary condition $U_g(\underline{u}_b) = \underline{u}_g$. Note that this does not depend on π. Given U_g, equations (90) – (91) can be solved for the distribution functions.

Given a functional form for the utility function, w, this system can be solved numerically. Figure 6 depicts the solution for the following parameterization: $w(c) = \sqrt{2c}$, $y = 10$, $d = 9$, $\theta_b = 0.9$, $\theta_g = 0.6$, $\mu_g = 0.3$. The left panel plots the equilibrium U_g, while the right panel shows the resource losses associated with imperfect insurance—specifically, the function $L(u_b)$ from (70).

Figure 6: Effect of varying competition



E.5.2 Type-Specific π

Since our proofs that F_h and F_l have no flat regions and F_h has no mass points immediately extend to the case when $\pi_l \neq \pi_h$, we omit them in the interest of brevity. Hence, we begin by analyzing the potential for mass point equilibria; that is, for $F_l(\cdot)$ to feature a mass point—to emerge when $\pi_l \neq \pi_h$.

Proposition 6. *Suppose $\pi_l < \pi_h$. Then $F_l(\cdot)$ does not have a mass point.*

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Proof. We prove a profitable deviation exists much as in the case when $\pi_l = \pi_h$. In particular, in any such equilibrium with a mass point, $\Pi_l = 0$ and the following inequalities must hold

$$\begin{aligned}
-\mu_h \left(1 - \pi_h + \pi_h F_l^- (\hat{u}_l)\right) \frac{v_h - c_h}{c_h - c_l} + \mu_l \left(1 - \pi_l + \pi_l F_l^- (\hat{u}_l)\right) &\leqslant 0 \\
\mu_h \left(1 - \pi_h + \pi_h F_l^+ (\hat{u}_l)\right) \frac{v_h - c_h}{c_h - c_l} - \mu_l \left(1 - \pi_l + \pi_l F_l^+ (\hat{u}_l)\right) &\leqslant 0.
\end{aligned}$$

Rearranging the above, we must have

$$\frac{1 - \pi_l + \pi_l F_l^- (\hat{u}_l)}{1 - \pi_h + \pi_h F_l^- (\hat{u}_l)} \leqslant \frac{\mu_h}{\mu_l} \frac{v_h - c_h}{c_h - c_l} \leqslant \frac{1 - \pi_l + \pi_l F_l^+ (\hat{u}_l)}{1 - \pi_h + \pi_h F_l^+ (\hat{u}_l)}. \tag{93}$$

Since $F_l^+ (\hat{u}_l) > F_l^- (\hat{u}_l)$ and $\pi_l < \pi_h$, then we must have that

$$\frac{1 - \pi_l + \pi_l F_l^- (\hat{u}_l)}{1 - \pi_h + \pi_h F_l^- (\hat{u}_l)} > \frac{1 - \pi_l + \pi_l F_l^+ (\hat{u}_l)}{1 - \pi_h + \pi_h F_l^+ (\hat{u}_l)},$$

which is a contradiction. ■

Proposition 7. *Suppose $\pi_l > \pi_h$. If a mass points exists, then $F_l(v_l) = 1$.*

Proof. First, it is immediate that a mass point cannot exist for any $u_l \neq v_l$. Hence, suppose by way of contradiction that there is a mass on v_l that is not full. Then either $F_l^- (v_l) > 0$ or $F_l^+ (v_l) < 1$. Since above and below v_l, the equilibrium features no mass points, the equilibrium must also satisfy the strict rank-preserving property. Let $S = \{(v_l, u_h)\}$ and note that S must have positive measure. Furthermore, the set S must be of the form $\{(v_l, u_h) : u_h \in [\underline{u}_h, \bar{u}_h]\}$. Note that we have, $\bar{u}_h > \underline{u}_h \geqslant c_h > v_l$.

Therefore, in a neighborhood around S, all equilibrium menus should be separating. As a result, they must satisfy the optimality condition with respect to u_l—for values of $u_l \in [v_l - \varepsilon, v_l + \varepsilon] \setminus \{v_l\}$ for small but positive ε (depending on whether mass is above or below v_l):

$$-\mu_l \left(1 - \pi_l + \pi_l F_l (u_l)\right) + \mu_l \pi_l f_l (u_l) (v_l - u_l) + \mu_h \left(1 - \pi_h + \pi_h F_h (u_h)\right) \frac{v_h - c_h}{c_h - c_l} = 0.$$

Using the SRP property,

$$-\mu_l \left(1 - \pi_l + \pi_l F_l (u_l)\right) + \mu_l \pi_l f_l (u_l) (v_l - u_l) + \mu_h \left(1 - \pi_h + \pi_h F_l (u_l)\right) \frac{v_h - c_h}{c_h - c_l} = 0.$$

Therefore, if positive mass is above v_l, we must have that

$$\mu_h \left(1 - \pi_h + \pi_h F_l (u_l)\right) \frac{v_h - c_h}{c_h - c_l} - \mu_l \left(1 - \pi_l + \pi_l F_l (u_l)\right) > 0,$$

and if it is below,

$$\mu_h \left(1 - \pi_h + \pi_h F_l (u_l)\right) \frac{v_h - c_h}{c_h - c_l} - \mu_l \left(1 - \pi_l + \pi_l F_l (u_l)\right) < 0.$$

From above, if mass point is to be an equilibrium property, the inequality (93) must hold:

$$\frac{1 - \pi_l + \pi_l F_l^- (v_l)}{1 - \pi_h + \pi_h F_l^- (v_l)} \leqslant \frac{\mu_h}{\mu_l} \frac{v_h - c_h}{c_h - c_l} \leqslant \frac{1 - \pi_l + \pi_l F_l^+ (v_l)}{1 - \pi_h + \pi_h F_l^+ (v_l)} < \frac{\pi_l}{\pi_h}. \tag{94}$$

Now suppose that $F_l^+(v_l) < 1$. Then, from the differential equation above,

$$F_l(u_l)\left[\mu_h\pi_h\frac{v_h-c_h}{c_h-c_l}-\pi_l\mu_l\right]-\mu_l\pi_l f_l(u_l)(u_l-v_l)+\mu_h(1-\pi_h)\frac{v_h-c_h}{c_h-c_l}-\mu_l(1-\pi_l)=0.$$

The general solution to the above differential equation is given by

$$F_l(u_l)=A_1(u_l-v_l)^{\frac{\mu_h\pi_h\frac{v_h-c_h}{c_h-c_l}-\pi_l\mu_l}{\mu_l\pi_l}}+A_2.$$

Since $\frac{\mu_h\pi_h\frac{v_h-c_h}{c_h-c_l}-\pi_l\mu_l}{\mu_l\pi_l}<0$ from (94), the above expression approaches either $\pm\infty$ as u_l approaches v_l from above. Hence, $F_l^+(v_l)<1$ cannot hold.

Now suppose that $F_l^-(v_l)>0$. Then, similar to above, we must have that

$$F_l(u_l)=A_1(v_l-u_l)^{\frac{\mu_h\pi_h\frac{v_h-c_h}{c_h-c_l}-\mu_l\pi_l}{\mu_l\pi_l}}+A_2.$$

As u_l converges to v_l, the above expression converges to ∞, which is in contradiction with $F_l^-(v_l)<1$. This proves the claim.

■

Proof of Proposition 4. We have already shown a mass point equilibrium, if it exists, must place full mass at v_l. Now, the worst menu in a mass point equilibrium (i.e., the one with the lowest u_h) must set $u_h=c_h$ (otherwise, lowering u_h strictly raises profits). By construction, a function F_h that satisfies (72) ensures equal profits at all points in the support. To rule out other deviations, consider the payoff from offering $u_l'=v_l-\varepsilon,\quad u_h'\in[\underline{u}_h,\bar{u}_h]$. The change in profits (per ε) satisfy

$$\mu_l(1-\pi_l)-(1-\pi_h+\pi_hF_h)\mu_h\frac{v_h-c_h}{c_h-c_l}=\left[1-\frac{(1-\pi_h+\pi_hF_h)}{(1-\pi_l)}\frac{\mu_h}{\mu_l}\frac{v_h-c_h}{c_h-c_l}\right]\mu_l(1-\pi_l).$$

It is sufficient to show that this is negative at the bottom, i.e., when $F_h=0$, which leads to

$$1-\frac{(1-\pi_h)}{(1-\pi_l)}\frac{\mu_h}{\mu_l}\frac{v_h-c_h}{c_h-c_l}<0\qquad\Rightarrow\qquad\frac{1-\pi_l}{1-\pi_h}<1-\phi.$$

To rule out equilibria without mass points, note that, in such an environment, the equilibrium is strictly rank-preserving, so there must be a worst menu, i.e., one with $F_l=F_h=0$. If it is a pooling menu, then it must offer $u_h=u_l=c_h$. In other words, $\Pi_l=v_l-c_h<0$. On the other hand, if it is a separating one, it must satisfy the FOC for u_l :

$$\frac{\pi_l f_l}{1-\pi_l+\pi_lF_l}\Pi_l=1-\left(\frac{1-\pi_h}{1-\pi_l}\right)(1-\phi_l)\quad<0\qquad\Rightarrow\qquad\Pi_l<0$$

i.e., the worst menu in an equilibrium without mass points must necessarily lose money on the low type. But then, the best menu must also lose money, because

$$\Pi_l(\bar{u}_l)=v_l-\bar{u}_l<v_l-\underline{u}_l<0.$$

Now, consider a deviation of the form $(\bar{u}_l-\varepsilon,\bar{u}_h)$ changes profits, relative to $(\bar{u}_l,\bar{u}_h)$, by

$$\mu_l-\mu_h\frac{v_h-c_h}{c_h-c_l}-\mu_lf_l\Pi_l(\bar{u}_l)=\mu_l\phi-f_l\Pi_l(\bar{u}_l)\quad>\quad0,$$

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yielding the desired contradiction. Thus, in a mass point equilibrium, the distribution of u_l is degenerate at v_l, i.e., buyers make zero profits from type-l sellers. A buyer can deviate and offer a lower u_l, but that brings higher profits only from the *captive* $l-$types at the expense of lower profits from both captive and noncaptive $h-$types. When the condition in part (1) of the proposition is satisfied, π_l is sufficiently high or equivalently, the fraction of captive $l-$types is too low to make such a deviation attractive.

E.5.3 Equilibrium with vertical differentiation

Here, we conjecture and characterize an equilibrium with vertical differentiation. We restrict attention to the region of the parameter space where both buyers offer separating contracts without cross-subsidization. First, note that the upper and lower bounds of the distributions of both buyers must coincide, i.e., the distributions of offers by both buyers have the same support. This then implies that F_l^2 has mass of α at its lowest point c_l. To see this, consider the equal profit condition for each buyer (recall that all ties are resolved in favor of buyer 1):

$$\begin{aligned}(1-\pi)(v_l-c_l) &= \Pi(\bar{u}_l,\bar{u}_h)\\ (1-\pi+\pi\alpha)(v_l-c_l+B) &= \Pi(\bar{u}_l,\bar{u}_h)+B.\end{aligned}$$

Solving, we obtain $\alpha=\frac{B}{B+v_l-c_l}$. Next, we posit that (i) $U_h^1(u_l)$ is strictly increasing everywhere in the support (ii) $U_h^2(u_l)=c_h$ for $u_l\in[c_l,c_l+s],\quad s\geqslant 0$. In the interval $(c_l+s,\bar{u}_l]$, $U_h^2(u_l)$ is strictly increasing. Formally, the distributions F_j^k satisfy the strict rank-preserving conditions

$$F_l^1(u_l) = F_h^1(U_h^1(u_l)) \qquad u_l\in[\underline{u}_l,\overline{u}_l] \tag{95}$$

$$F_l^2(u_l) = F_h^2(U_h^2(u_l)) \qquad u_l\in(c_{l+s},\overline{u}_l]. \tag{96}$$

The optimality conditions for u_l and u_h for the two buyers yield:

$$\frac{\pi f_l^2(u_l)}{1-\pi+\pi F_l^2(u_l)}\Pi_l^1(u_l) = 1-\frac{\mu_h}{\mu_l}\left(\frac{1-\pi+\pi F_h^2\left(U_h^1(u_l)\right)}{1-\pi+\pi F_l^2(u_l)}\right)\frac{v_h-c_h}{c_h-c_l} \tag{97}$$

$$\frac{\pi f_h^2(u_h)}{1-\pi+\pi F_h^2\left(U_h^1(u_l)\right)}\Pi_h^1\left(u_l,U_h^2(u_l)\right) = \frac{v_h-c_l}{c_h-c_l} \tag{98}$$

$$\frac{\pi f_l^1(u_l)}{1-\pi+\pi F_l^1(u_l)}(v_l-u_l) = 1-\frac{\mu_h}{\mu_l}\left(\frac{1-\pi+\pi F_h^1\left(U_h^2(u_l)\right)}{1-\pi+\pi F_l^1(u_l)}\right)\frac{v_h-c_h}{c_h-c_l} \tag{99}$$

$$\frac{\pi f_h^1(u_h)}{1-\pi+\pi F_h^1\left(U_h^2(u_l)\right)}\Pi_h^2\left(u_l,U_h^2(u_l)\right) = \frac{v_h-c_l}{c_h-c_l}. \tag{100}$$

This system of equations (95) − (100), along with the boundary conditions

$$\begin{aligned}F_l^1(c_l) &= F_h^1(c_h)=0\\ F_l^2(c_l) &= \alpha\\ F_l^1(\overline{u}_l) &= F_l^2(\overline{u}_l)=1\\ F_h^1(\overline{u}_h) &= F_h^2(\overline{u}_h)=1\\ (1-\pi)(v_l-c_l) &= \left(1-\pi+\pi F_l^1(c_l+s)\right)(v_l-c_l-s)+(1-\pi)\Pi_h(c_l+s,c_h)\end{aligned}$$

characterize the six unknown functions $F_l^1, F_l^2, F_h^1, F_h^2, U_h^1$, and U_h^2.

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E.5.4 Proofs for Extension to N Types

Proof of Lemma 14. This proof is a direct extension of the proof of Lemma 1, and hence is omitted for brevity.

Proof of Proposition 5. To show the strict rank-preserving property, we first show that F_j's are continuous and strictly increasing. The argument for this claim is inductive.

Step 1: F_N is strictly increasing and continuous.

F_N *is strictly increasing.* Suppose, toward a contradiction, that there is an interval $[u'_N, u''_N]$ where F_N is constant and takes a value between 0 and 1. Without loss of generality, we can assume that u''_N belongs to some contract that is offered in equilibrium. Let one such menu be given by $\mathbf{u}'' = (u''_1, \cdots, u''_N)$. Given our assumption that the equilibrium is separating, this menu must maximize $\sum_{i=1}^{N} \mu_i (1 - \pi + \pi F_i(u_i)) \Pi_i(u_{i-1}, u_i)$ over the set of menus that are subject to the participation constraints. Now consider a menu given by $\left(u''_1, \cdots, u''_{N-1}, u''_N - \varepsilon\right)$ for a small ε. Since $u''_N > u'_N \geqslant c_N$, this menu satisfies the participation constraint. Moreover, this menu keeps the fraction of noncaptive N types constant while increasing profits per N-th type, thus yielding higher profits, a contradiction.

F_N *is continuous.* Suppose, toward a contradiction, that F_N has a mass point at $\hat{u}_N$. Let $\mathbf{u} = (u_1, \cdots, u_{N-1}, \hat{u}_N)$ be an arbitrary equilibrium menu with its N-th element given by $\hat{u}_N$. Note that we must have $\Pi_N(u_{N-1}, \hat{u}_N) \leqslant 0$ and $\hat{u}_N = c_N$. The fact that $\Pi_N(u_{N-1}, \hat{u}_N) \leqslant 0$ is immediate, since otherwise a small increase in $\hat{u}_N$ would attain a higher level of profits. Additionally, if $\hat{u}_N > c_N$, then a small decrease in $\hat{u}_N$ would attain higher profits. Such a change increases profits because either $\Pi_N < 0$—in which case this change decreases the probability that an N type accepts the offer discretely—or $\Pi_N = 0$—in which case this change makes profits per N type strictly positive.

Non-positivity of profits, together with $\hat{u}_N = c_N$, implies that

$$v_N - \frac{v_N - c_{N-1}}{c_N - c_{N-1}} c_N + \frac{v_N - c_N}{c_N - c_{N-1}} u_{N-1} \leqslant 0 \Rightarrow \frac{v_N - c_N}{c_N - c_{N-1}} u_{N-1} \leqslant \frac{v_N - c_N}{c_N - c_{N-1}} c_{N-1} \Rightarrow u_{N-1} \leqslant c_{N-1}.$$

This inequality, together with the participation constraint, $c_{N-1} \leqslant u_{N-1}$, implies that u_{N-1} must equal c_{N-1} and $\Pi_N = 0$. That is, any menu $\mathbf{u}$ with $\hat{u}_N$ as its N-th element must also satisfy $u_{N-1} = c_{N-1}$, so that F_{N-1} must also have a mass point at c_{N-1}. Repetition of this argument implies that any menu containing a mass point at $\hat{u}_N$ must also satisfy $u_j = c_j$, and thus F_j must have a mass point at c_j. However, then a small increase in u_1 from $u_1 = c_1$ must increase profits, as F_1 puts a mass at c_1 and profits from type 1 sellers are positive. This yields the necessary contradiction.

Step 2: If $\{F_k\}_{k=j+1}^{N}$ are strictly increasing and continuous, then F_j must have the same properties.

To prove this claim, we first prove the following lemma:

Lemma 17. *Suppose that, for some $j \leqslant N-1$, the distributions $\{F_k\}_{k=j}^{N}$ are continuous and strictly increasing. Then there exists a sequence of strictly increasing and continuous functions $\{U_{k,j}(u_j)\}_{k=j+1}^{N}$ such that for any menu $\hat{\mathbf{u}}$ offered in equilibrium with its j-th element given by $\hat{u}_j$, $(\hat{u}_{j+1}, \cdots, \hat{u}_N) = (U_{j+1,j}(\hat{u}_j), \cdots, U_{N,j}(\hat{u}_j))$.*

Proof. We prove this claim by induction. For any value of u_{N-1}, let $U_N^+(u_{N-1})$ be the set of values of u_N such that equilibrium menus exist with $(N-1)$-th and N-th elements given by (u_{N-1}, u_N).

We first show that $U_N^+(u_{N-1})$ is a strictly increasing function. Using exactly the same arguments as in the two-type case, it is straightforward to show that: (*i*) $U_N^+(u_{N-1})$ must be a strictly increasing correspondence; and (*ii*) if $u, u' \in U_N^+(u_{N-1})$, then $[u, u'] \subseteq U_N^+(u_{N-1})$. These results are direct implications of strict supermodularity of the function $\mu_N(1 - \pi + \pi F_N(u_N))\Pi_N(u_{N-1}, u_N)$ and the strict monotonicity of F_N.

Now suppose that for some $\hat{u}_{N-1}$, $U_N^+(\hat{u}_{N-1})$ is a correspondence and so contains an interval given by $[u', u'']$. Then

$$\Pr(u_{N-1} = \hat{u}_{N-1}) = \int_{\{(u_1,\cdots,u_{N-2},\hat{u}_{N-1},u_N)\in Supp(\Phi)\}} d\Phi \geqslant F_N(u'') - F_N(u') > 0,$$

where the last inequality follows from the fact that F_N is strictly increasing. This inequality implies that F_{N-1} has a mass point at $\hat{u}_{N-1}$, in contradiction with the assumption that F_{N-1} is continuous. Hence, U_N^+ must be a single-valued function.

One can also adapt our arguments from the two-type case to show that $U_N^+(u_{N-1})$ is strictly increasing. If it were constant on an interval, then F_N must have a mass point, contradicting the continuity of F_N. Thus, $U_N^+(u_{N-1})$ is a strictly increasing function and we may write profits from the N-th type as function of u_{N-1} only. Let this function be given by $\Pi_N^+(u_{N-1})$.

Next, let $U_{N-1}^+(u_{N-2})$ be defined in a similar fashion as above. Since the profit function

$$\mu_{N-1}(1 - \pi + \pi F_{N-1}(u_{N-1}))\Pi_{N-1}(u_{N-2}, u_{N-1}) + \Pi_N^+(u_{N-1})$$

is strictly supermodular and F_{N-1} and F_{N-2} are strictly increasing and continuous, U_{N-1}^+ must be a strictly increasing, single-valued function. Exact repetition of this argument implies that for all $k \in \{j, \ldots, N-1\}$, U_j^+ is a strictly increasing function. Therefore, we must have that

$$U_{k,j}(\hat{u}_j) = U_k^+\left(U_{k-1}^+\left(\cdots\left(U_{j+1}^+(\hat{u}_j)\right)\right)\right)$$

for all $k \in \{j+1, \ldots, N\}$, and this concludes the proof. ■

We now return to proving step 2 of the induction argument.

F_j *is strictly increasing.* Suppose, by way of contradiction, that F_j has a flat over an interval $[u_j', u_j'']$. Much as in Lemma 5, we prove that if F_j is flat on the interval $[u_j', u_j'']$, then the marginal benefit of delivering one additional unit of surplus to type $j+1$ (incorporating the impact on all types $i > j+1$) changes with $u_j \in [u_j', u_j'']$. This fact allows us to show alternative menus with higher levels of profits than the conjectured equilibrium level must exist.

To see this, first let $U_{j+1}^+(u_j)$ be the correspondence defined in the proof of Lemma 17. By our induction assumption and Lemma 17, profits from types $\{j+1, \ldots, N\}$ can be written as

$$\mu_{j+1}(1 - \pi + \pi F_{j+1}(u_{j+1}))\Pi_{j+1}(u_j, u_{j+1}) + \Pi_{j+2}^+(u_{j+1}),$$

where $\Pi_{j+2}^+(u_{j+1})$ are equilibrium profits constructed by applying $U_{k,j+1}$ as defined in Lemma 17. Note that these profits are strictly supermodular in (u_j, u_{j+1}), and, as a result, $U_{j+1}^+(u_j)$ is a strictly increasing correspondence. Additionally, since F_j is flat over the interval $[u_j', u_j'']$, we must have that $U_{j+1}^+(u_j')$ and $U_{j+1}^+(u_j'')$ must have a common element (as in the proof of Lemma 5). Let $\overline{u}_{j+1}$ be this common element.

Let $\mathbf{u}'$ be an equilibrium menu with j-th element given by u_j' and $(j+1)$-th element given by $\overline{u}_{j+1}$ and $\mathbf{u}''$ be an equilibrium menu with j-th element given by u_j'' and $j+1$-th element given by $\overline{u}_{j+1}$. Note that a perturbation of $\mathbf{u}'$ that increases u_j' by a small amount must not increase profits. Similarly, a perturbation of $\mathbf{u}''$ that decreases u_j'' by a small amount must not increase profits. Since F_j is flat on $[u_j', u_j'']$, non-positivity of these two perturbations imply

$$-\mu_j F_j(u_j')\frac{v_j - c_{j-1}}{c_j - c_{j-1}} + \mu_{j+1}F_{j+1}(\overline{u}_{j+1})\frac{v_{j+1} - c_{j+1}}{c_{j+1} - c_j} = 0. \quad (101)$$

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As a consequence, profits obtained from any menu $\hat{\mathbf{u}}$, which is the same as $\mathbf{u}'$ except at its j-th element and has j-th element equal to $u_j \in [u_j', u_j'']$, must yield the same profits as $\mathbf{u}'$.

We now show that a perturbation from some such $\hat{\mathbf{u}}$ must strictly increase profits. In particular, consider a perturbation from $\hat{\mathbf{u}}$ that increases $u_{j+1} = \bar{u}_{j+1}$ by a small amount, ε. Since F_{j+1} is strictly increasing and continuous, the change in profits from this perturbation is given by

$$\mu_{j+1} f_{j+1}\left(\overline{u}_{j+1}\right) \Pi_{j+1}\left(u_j, \overline{u}_{j+1}\right) + \mu_{j+1}\left(1-\pi+\pi F_{j+1}\left(\overline{u}_{j+1}\right)\right) \frac{\nu_{j+1}-c_j}{c_{j+1}-c_j} + \frac{d}{du_{j+1}} \Pi_{j+2}^{+}\left(\overline{u}_{j+1}\right). \tag{102}$$

Since $f_{j+1}\left(\overline{u}_{j+1}\right) > 0$ and Π_{j+1} is linear in u_j, the expression in (102) must be nonzero for some $u_j \in (u_j', u_j'')$. This implies some menu can strictly raise profits above the conjectured equilibrium level and is a contradiction. Thus, F_j cannot have a flat.

F_j *is continuous.* Now suppose that F_j has a discontinuity at $\hat{u}_j$. As in step 1, it must be that $\Pi_j\left(\hat{u}_{j-1}, \hat{u}_j\right) \leqslant 0$. There are two possibilities: $\hat{u}_j = c_j$ or $\hat{u}_j > c_j$. If $\hat{u}_j = c_j$, then a straightforward adaptation of the argument in step 1—where we proved F_N is continuous—can be applied to yield a contradiction. Hence, consider the second case with $\hat{u}_j > c_j$. Notice immediately that $\Pi_j\left(\hat{u}_{j-1}, \hat{u}_j\right)$ must equal zero, since otherwise a small decrease in $\hat{u}_j$ would strictly increase profits. Since there is a unique value $\hat{u}_{j-1}$ such that $\Pi_j\left(\hat{u}_{j-1}, \hat{u}_j\right) = 0$, if F_j has a mass point at $\hat{u}_j$, F_{j-1} must also have a mass point at some $\hat{u}_{j-1}$. Repeating this argument implies that F_1 must have a mass point, and this mass point must be at ν_1, since $u_1 = \nu_1$ is the unique value such that $\Pi_1(u_1) = 0$.

Let $\mathbf{u} = \left(\nu_1, \ldots \hat{u}_{j-1}, \hat{u}_j, u_{j+1}, U_{j+2,j+1}\left(u_{j+1}\right), \ldots, U_{N,j+1}\left(u_{j+1}\right)\right)$. Since the distribution functions $F_{j+1}, \ldots, F_N$ have no mass points, $U_{j+1}^{+}(\hat{u}_j) = [u_{j+1}', u_{j+1}'']$ for some values u_{j+1}' and u_{j+1}''.

Let $1 \leqslant k \leqslant j$ be the highest index for which $\phi_k \neq 0$; recall, by assumption, $\phi_1 \neq 0$ so that $k \geqslant 1$. Now consider two different perturbations from $\mathbf{u}$, where we perturb elements k through j according to

$$\begin{aligned}
\mathbf{u}^{-} &= \left(\nu_1, \ldots, \hat{u}_{k-1}, \hat{u}_k - \epsilon, \ldots, u_j' - \epsilon, u_{j+1}', U_{j+2,j+1}\left(u_{j+1}'\right), \ldots, U_{N,j+1}\left(u_{j+1}'\right)\right), \\
\mathbf{u}^{+} &= \left(\nu_1, \ldots, \hat{u}_{k-1}, \hat{u}_k + \epsilon, \ldots, u_j' + \epsilon, u_{j+1}'', U_{j+2,j+1}\left(u_{j+1}''\right), \ldots, U_{N,j+1}\left(u_{j+1}''\right)\right).
\end{aligned}$$

For small ϵ, the change in the profits from the above perturbations are, respectively, given by

$$\begin{aligned}
&\mu_k(1-\pi+\pi F_k^{-}(\hat{u}_k))\frac{\nu_k - c_{k-1}}{c_k - c_{k-1}} + \mu_{k+1}(1-\pi+\pi F_{k+1}^{-}(\hat{u}_{k+1})) + \cdots + \mu_j(1-\pi+\pi F_j^{-}(\hat{u}_j)) \\
&\qquad -\mu_{j+1}(1-\pi+\pi F_{j+1}^{-}(u_{j+1}'))\frac{\nu_{j+1}-c_{j+1}}{c_{j+1}-c_j}, \\
&-\mu_k(1-\pi+\pi F_k^{+}(\hat{u}_k))\frac{\nu_k - c_{k-1}}{c_k - c_{k-1}} - \mu_{k+1}(1-\pi+\pi F_{k+1}^{+}(\hat{u}_{k+1})) - \cdots - \mu_j(1-\pi+\pi F_j^{+}(\hat{u}_j)) \\
&\qquad +\mu_{j+1}(1-\pi+\pi F_{j+1}^{-}(u_{j+1}''))\frac{\nu_{j+1}-c_{j+1}}{c_{j+1}-c_j}.
\end{aligned}$$

Since the distributions F_i are well behaved above and below each $\hat{u}_i$, the strict rank-preserving property implies $F_i^{-}(\hat{u}_i) = F_{j+1}(u_{j+1}')$, and $F_i^{+}(\hat{u}_i) = F_{j+1}(u_{j+1}'')$ for all values of $i \leqslant j$. We may then write the change in profits from these perturbations, respectively, as

$$\begin{gathered}
(1-\pi+\pi F_k^{-}(\hat{u}_k))\sum_{i=k}^{j}\mu_i\phi_i, \\
-(1-\pi+\pi F_k^{+}(\hat{u}_k))\sum_{i=k}^{j}\mu_i\phi_i.
\end{gathered}$$

Since k is the highest index below j for which $\phi_k \neq 0$, one of the above expressions must be positive.

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Therefore, one of the constructed menus increases profits, yielding a contradiction. The claim that equilibrium is strictly rank-preserving then follows immediately from Lemma 17. ■

Proof of Lemma 15. The monopsonist maximizes

$$\mu_1 (\nu_1 - u_1) + \sum_{i=2}^{N} \mu_i \left[\nu_i - \frac{\nu_i - c_{i-1}}{c_i - c_{i-1}} u_i + \frac{\nu_i - c_i}{c_i - c_{i-1}} u_{i-1}\right] = \sum_{i=1}^{N} \mu_i (\nu_i - \phi_i u_i)$$

subject to the monotonicity constraint

$$1 \geqslant \frac{u_n - u_{n-1}}{c_n - c_{n-1}} \geqslant \cdots \geqslant \frac{u_{i+1} - u_i}{c_{i+1} - c_i} \geqslant \frac{u_i - u_{i-1}}{c_i - c_{i-1}} ... > 0. \tag{103}$$

Given the linearity in payoffs and constraints, the solution to this problem is a single price offer, i.e., $u_i = c_J$, $i \leqslant J$ and $u_i = c_i$ for $i > J$ for some $J \in \{1, 2...N\}$; see arguments in Myerson (1985) and Samuelson (1984). To see why J must be the *largest* integer such that $\sum_{i=1}^{J-1} \mu_i \phi_i < 0$, suppose otherwise, i.e., $\exists\ k < J$ such that $\sum_{i=1}^{k-1} \mu_i \phi_i < 0$ and the monopsonist sets $u_i = c_k$ for $i \leqslant k$ and $u_i = c_i$ for $i > k$. Then, a deviation which increases all u_i for $i < J$ by ε changes profits by $-\varepsilon \sum_{i=1}^{J-1} \mu_i \phi_i > 0$. ■

Proof of Lemma 16. To show that the best equilibrium menu satisfies $u_i = u_J$ for $i < J$, suppose by way of contradiction that for some $i < J$, $u_i < u_J$. The monotonicity constraint implies $u_J > u_{J-1}$; if $u_J = u_{J-1}$, then we must have $u_i = u_{i-1}$ for all $i < J$. Now, consider an alternative menu that increases all the utilities of types below J by ε. The probability of trade with any type does not change (since this is already the best menu), the change in profits is given by $-\varepsilon \sum_{i=1}^{J-1} \mu_i \phi_i$, which is strictly positive by the definition of J in (89).

To show that the worst equilibrium menu satisfies $u_i = c_i$ for $i \geqslant J$, suppose by way of contradiction that $u_{J+k} > c_{J+k}$ for some $k \geqslant 0$. This inequality, together with repeated application of the monotonicity constraint, implies that $u_i > c_i$ for all $i \leqslant J + k$. Now consider an alternative menu that lowers the utility of all types below and including $J + k$ by ε. This does not change the probability of trade, as the original menu is the worst menu. However, the change in profits from captive types is $\varepsilon \sum_{i=1}^{J+k} \mu_i \phi_i$, which is positive by the definition of J in (89). ■

The Solution to the System of ODEs in (88). The general solution to this system of equations depends on the sign of the profits from the lowest types, $\nu_1 - u_1$. From (87), this profit is positive when $\phi_1 > 0$, and negative when $\phi_1 < 0$. In what follows, we assume that the sequence $\gamma_i = \frac{\nu_i - c_{i-1}}{c_i - c_{i-1}} \frac{\phi_1}{\phi_j}$ takes on different values for all $i \geqslant 2$, i.e., $\gamma_i \neq \gamma_j$.[16] We thus have the following general solution:

$$U_i = \sum_{k=0}^{i} a_{k,i} \left(|\nu_1 - u_1|\right)^{\gamma_k}$$

with

$$\gamma_0 = 0, \gamma_1 = 1,$$

where

$$\begin{aligned} a_{0,i} &= \frac{\nu_i (c_i - c_{i-1})}{\nu_i - c_{i-1}} + \frac{\nu_i - c_i}{\nu_i - c_{i-1}} a_{0,i-1} \\ a_{k,i} &= \frac{\nu_i - c_i}{\nu_i - c_{i-1}} \frac{\gamma_i}{\gamma_i - \gamma_k} a_{k,i-1} \end{aligned}$$

[16] While it is possible to provide the general solution of the ODEs, this assumption greatly simplifies the formulation.

with

$$
\begin{aligned}
a_{0,1} &= \nu_1 \\
a_{1,1} &= \mathbf{sgn}(\nu_1 - u_1),
\end{aligned}
$$

where **sgn** is 1 if its argument is positive and -1 when its argument is negative.

In the above formulation, the variables $\{a_{i,i}\}_{i=2}^{N}$ are unknown and have to be determined by the boundary conditions in Lemma 16. To do this, for any value of $\underline{u}_1 = \min Supp(F_1)$, we can use equation (87) to solve for F_1, with the boundary condition for $\underline{u}_1$. We can then find the value of $\overline{u}_1$, i.e., the upper bound of the support of F_1, using $F_1(\overline{u}_1) = 1$. We refer to this value as $\tilde{u}_1(\underline{u}_1)$ as a function of $\underline{u}_1$. The boundary conditions then are given by:

$$
\begin{aligned}
&U_J(\underline{u}_1) = c_J, \cdots, U_N(\underline{u}_1) = c_N \\
&U_2(\tilde{u}_1(\underline{u}_1)) = \tilde{u}_1(\underline{u}_1), \cdots, U_J(\tilde{u}_1(\underline{u}_1)) = \tilde{u}_1(\underline{u}_1)
\end{aligned}
$$

This is a system of $N - J + 1 + J - 1 = N$ equations with N unknowns given by $a_{i,i}{}_{i=2}^{N}$ and $\underline{u}_1$. Solving this system of equations determines the equilibrium.

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